An imperfect storm: Fat-tailed tropical cyclone damages, insurance, and climate policy

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**Abstract**

We perform two tests that estimate the mass of the upper tail of the distribution of aggregate US tropical cyclone damages. Both tests reject the hypothesis that the distribution of damages is thin tailed at the 95% confidence level, even after correcting for inflation and growth in population and per capita income. Our point estimates of the shape parameter of the damage distribution indicate that the distribution has finite mean, but infinite variance.

In the second part of the paper, we develop a microfoundations model of insurance and storm size that generates a fat tail in aggregate tropical cyclone damages. The distribution of the number of properties within a random geographical area that lies in the path of a tropical cyclone is shown to drive fat tailed storm damages, and we confirm that the distribution of coastal city population is fat tailed in the US. We show empirically and theoretically that other random variation, such as the distribution of storm strength and the distribution of damages across individual properties, does not generate a fat tail. We consider policy options such as climate change abatement, policies which encourage adaptation, reducing subsidies for coastal development, and disaster relief policies, which distort insurance markets. Such policies can reduce the thickness of the tail, but do not affect the shape parameter or the existence of the fat tail.

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1. Introduction

Hurricanes are the most costly natural disasters in the US (Smith and Katz, 2013). Indeed, several recent storms were catastrophic, causing property damage, loss of life, dislocations, and suppression of economic activity. For example, Hurricanes Sandy and Katrina caused property damage of approximately $51 and $96 billion (2012 dollars), respectively. Although some hurricanes are catastrophic, many tropical cyclones impose much smaller costs: the median property damage for tropical cyclones that make landfall is only $870 million. Hence, one might suspect that the distribution of tropical cyclone damages is fat tailed. Non-technically, a fat upper-tailed distribution exhibits a slow rate of decay in the probability of...
increasingly large observations, and so extreme realizations are much more likely than for a thin-tailed distribution, such as the normal distribution.

We first provide empirical evidence that the distribution of tropical cyclone damages is indeed fat tailed, even after controlling for inflation and growth in population and per capita income. Next, we develop a microfoundations model that generates a fat-tailed aggregate damage distribution, and show how public policies affect the upper tail of the distribution.

We perform two tests to estimate the thickness of the tail of the damage distribution. The first test estimates the parameters of the generalized Pareto distribution (GPD) using maximum likelihood. The GPD nests both fat and thin tails and is a commonly used test for fat tails (Brazouskas and Kleefeld, 2009). The second test estimates the slope of the mean excess function: the expectation conditional on a realization being larger than a threshold. The slope of the mean excess as a function of the threshold is positive if and only if the distribution is fat tailed (Ghosh, 2010).

The two tests produce similar estimates of the shape parameter of the damage distribution in the range of 0.66–0.8, rejecting the hypothesis that the distribution of damages is thin tailed at the 95% confidence level. The point estimates of the shape parameter suggest that the damage distribution is fat tailed with finite mean but infinite variance.2

A fat-tailed storm damage distribution poses a number of challenges. Mean storm damages, an important consideration in insurance pricing, are more difficult to estimate when damages are drawn from a fat-tailed distribution, because extreme values (e.g., catastrophic storms), for which relatively few observations exist, exert strong influence on the estimate of the mean. Similarly, the expected damage from a catastrophic storm (e.g., a once-in-a-hundred-years storm),3 is also more difficult to estimate, meaning that regulators must require insurers to hold larger minimum levels of reserve capital. It is also well known that insurance firms must hold more costly reserves when the estimate of the damages from a catastrophic storm is uncertain. The cost of holding reserves is passed on, in the form of higher premiums, to policyholders, who may respond by not fully insuring (Kousky and Cooke, 2012). Identifying welfare-enhancing policies in such an environment requires an empirical and theoretical understanding of the determinants of the existence and mass of the fat tail in the damage distribution.

Hurricanes that cause catastrophic damages feature extraordinary storm intensity (e.g., Andrew, which had a maximum sustained wind speed of 170 mph), make landfall in areas with an extraordinarily large population and property base (e.g., Sandy, which struck the New York metropolitan area), and cause extraordinary damage to individual properties (e.g., Kousky and Michel-Kerjan, 2015 find 63% of flood insurance claims greater than or equal to 95% of the insured value between 1978 and 2012 occurred in 2005, presumably due to Hurricane Katrina). Aggregate damages are a function of each of these random variables. A fat tail in the distribution of one or more of these random variables can imply a fat-tailed damage distribution.

One possible cause of fat-tailed storm damages is the distribution of storm intensity. However, we show empirically that storm wind speed is thin tailed.4 This result is intuitive: the expected wind speed conditional on exceeding a threshold should decline with the threshold, because the extra energy required to generate and sustain larger storms is not readily available. Our results show that the wind speed distribution cannot cause a fat tail in the damage distribution.

Aggregate damages are the sum of individual property damages. Because property damages are bounded and therefore have finite variance, the central limit theorems apply. The sum of property damages becomes approximately normally distributed (thin tailed) as the number of properties becomes large; random variation in the individual damage across properties cannot be the source of the fat tail in the damage distribution.

A third possibility is that the distribution of coastal property is fat tailed. We use coastal population data as a proxy for property and test the coastal population distribution for fat tails, using Census data on the population of incorporated places in coastal counties. We find that the coastal population distribution is fat tailed, with a shape parameter in the range of 0.6 to 0.8, very close to the shape parameter of the aggregate damage distribution. Our empirical results indicate that a determinant of the fat-tailed damage distribution is a fat tail in the geographic distribution of property. Most storms intersect geographic areas with little or no property. However, storms intersect areas with large amounts of property more often than expected given a normal distribution, causing the fat tail in the damage distribution.

Having demonstrated empirically that storm damages are fat tailed, we present a model of homeowner behavior that links policies to the mass of the upper tail of the storm damage distribution. In the model, households purchase coastal and inland property at constant marginal cost. The cost of property insurance raises the total cost of ownership for coastal properties, thus discouraging development.5 A disaster relief agency exists, which reimburses a fraction of household losses.

The total coastal property at risk as determined by household choices is distributed geographically into population centers. Following the empirical results, the size distribution of the population centers has a fat tail. Finally, a model of storm size,
wind speed, and adaptations (e.g. building codes) determines the area of damage. In the model, a storm of random (thin tailed) size and wind speed may intersect a population center of random (fat tailed) size, in which case individual properties experience random, bounded damages.

We show that, as the number of potentially affected properties becomes large, the damage distribution converges to a distribution that is approximately fat tailed, with a tail index equal to the tail index of the property distribution, which is consistent with our close empirical estimates of the shape parameters for damage and coastal population. Although aggregate damages are the sum of individual damages with finite variance, the aggregate damage distribution is not normal, because the number of terms in the sum (the number of properties in the population center) is random and fat tailed.

The costs of fat-tailed damages motivate public policies to reduce the mass of the upper tail of the damage distribution. In the model, the distribution of storm damages is a function of several policies, including climate policy (which affects the frequency of large storms), disaster relief and development subsidies (which affect the total property at risk), and adaptations (which affect the minimum wind speed that causes damage). Each of these policies affects the damage from a catastrophic storm in different ways.

One important class of policies is disaster relief, coastal infrastructure projects, insurance subsidies, and other subsidies to coastal development. For example, the Federal Emergency Management Agency (FEMA) provides relief from uninsured storm in different ways. (which affect the minimum wind speed that causes damage). Each of these policies affects the damage from tropical cyclones. The rise in coastal development has led to concern that coastal development subsidies increase cyclone damage (Bunby, 2006; Bagstada et al., 2007). We show that moral hazard created by coastal development subsidies adds to the total clones. The rise in coastal development has led to concern that coastal development subsidies increase cyclone damage in presidentially declared disaster areas. This intervention is meant to reduce the welfare losses from tropical cyclones. The rise in coastal development has led to concern that coastal development subsidies increase cyclone damage (Bunby, 2006; Bagstada et al., 2007). We show that moral hazard created by coastal development subsidies adds to the total damage. Each of these policies affects the damage from a catastrophic storm in different ways.

Adaptations such as stronger building codes affect the damage distribution. Hurricane strength declines with the distance from the eye of the storm (Holland, 1980). If a random storm path intersects a population center away from the eye of the storm, then little or no damage will result if building codes are sufficiently strong. We show that an increase in the strength of adaptations reduces the geographic area of damage in two ways. First, adaptations, such as stronger building codes, reduce the radius of winds that are strong enough to cause damage, as the wind speed beyond a certain distance from the eye is no longer strong enough to cause damage. Second, adaptations decrease the time from landfall during which the tropical cyclone is sufficiently strong to cause damages. The maximum wind speed declines over land, reaching the point where adaptations are strong enough to prevent damage more quickly. In turn, the reduction in the geographic area of damage implies that more storms miss population centers and cause no damage. Thus, adaptations reduce the probability of a catastrophic storm.

Climate change abatement is another policy lever that affects the aggregate damage distribution. Some research suggests that a positive link exists between greenhouse gas emissions and tropical cyclone frequency (Holland and Webster, 2007; Mann and Emanuel, 2006; Emanuel, 2005). However, there is not a consensus on this point, as other studies fail to support that conclusion (Vecchi et al., 2008; Vecchi and Knutson, 2008; Knutson et al., 2008). Terry (2017) conducts Monte Carlo simulations using draws from thin-tailed distributions of predicted hurricane frequencies, sea levels, vulnerability-weighted population, and vulnerability-weighted per capita income, and finds that hurricane damages in the United States will increase dramatically by 2075, outpacing GDP growth, due to the interacting effects of climate change and projected increases in coastal development. One recent study finds that the number of intense storms (category 4 or 5 hurricanes) may double by the end of the 21st century, even as the overall frequency of tropical cyclones decreases (Bender et al., 2010), suggesting that changing climate may primarily affect the upper tail of the distribution of storm size.

Limiting climate change limits the energy available for storm formation. We show that climate change abatement reduces the wind speed of storms, moderating the size and duration of storms. With abatement, the geographic area of damage decreases, reducing the probability that a storm will impact a large population center. Thus abatement makes catastrophic storms even more rare.

Finally, we show that none of the policies we consider affect the tail index or the existence of the fat tail. While the policies can reduce the mass of the tail (and thus the mean damage and the damage from catastrophic storms), the policies cannot make a fat-tailed distribution thin tailed. Specifically, the model predicts that regardless of policy differences (e.g. across less or more developed countries or regions), the right tail of the damage distribution is asymptotically equivalent to the same Pareto distribution.

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6 For the GPD distribution, the tail index is the inverse of the shape parameter.
7 Populations in coastal communities at risk of storm damages are growing more rapidly than populations in the rest of the United States (Blake et al., 2011).
8 Pielke et al. (2008), fail to identify an observable linear trend in the mean damage associated with each storm making landfall in the United States from 1900-2008 after controlling for population growth and inflation. In contrast, our focus is on changes in other moments, such as the risk of catastrophic storms.
9 Excluding the extreme case of a policy which results in adaptations that are sufficiently strong to prevent any damages from tropical cyclones.
Prior literature

Our results relate to a number of strands of literature. First, our empirical results complement a growing body of literature showing that the distribution of total damages from natural disasters is often fat tailed. For example, Kousky and Cooke (2009) find evidence of fat-tailed aggregate damages for several varieties of natural disasters, including crop indemnities and National Flood Insurance claims.

Fat-tailed damages can also arise in the context of climate change from uncertain parameters that have a nonlinear effect on climate damages. Weitzman (2009) shows that fat-tailed damages may result in expected utility, and therefore willingness to pay to avoid damage risk, which is unbounded (the ‘dismal theorem’). The importance of the dismal theorem for climate policy is a subject of theoretical debate (Weitzman, 2009; Nordhaus, 2011). Nonetheless, Kelly and Tan (2015) show that fat-tailed climate damages can be important quantitatively, supporting the importance of studying fat-tailed damages in other contexts, such as tropical cyclone damages.

Our results relate to a large literature which explores how institutional, political, and other national-level features affect disaster impacts using multi-country panel data (e.g. Kahn, 2005; Hideki and Skidmore, 2007; Kellenberg and Mobarak, 2008). These studies focus on the relationship between GDP and mean cyclone damages. In contrast, in our work the fat-tailed geographic distribution of property is the primary determinant of the right tail of the distribution of cyclone damages.

While we are not aware of any other attempt to create a microfoundations model of fat-tailed cyclone damages, previous work shows how the mass of the tail increases through correlation of damages across properties. For example, Kousky and Cooke (2012) show that the mass of the tail increases when either small correlations exist between individual property damages or when extreme losses are correlated. While spatial correlation is important (and indeed our fat-tailed property distribution implies spatial correlation of individual damages), our microfoundations approach allows for analysis of the effect of policies on the mass of the tail.

Our empirical results relate to a literature which finds considerable evidence that the distribution of city population in the US is fat tailed (see for example, Gabaix, 1999; Gabaix and Ibragimov, 2011; González-Val et al., 2015). Gabaix and Ibragimov (2011) and others find the exponent of the Pareto distribution is close to one (known as Zipf’s law) for US metropolitan areas above a minimum population. However, González-Val et al. (2015) considers the GDP and finds a range of potential exponents. Our results using only coastal cities in 2010 also rejects thin tails, albeit with a somewhat smaller exponent. In particular, these studies find an exponent which implies the mean and variance are infinite, whereas our results indicate a finite mean and finite variance.

Our theoretical model relates to a large literature on coastal development subsidies and disaster assistance. Bagstada et al. (2007) review many types of coastal development subsidies and the incentives they create. Sutter (2007) finds that many states subsidize coastal insurance by establishing “wind/beach pools” in which the government offers subsidized insurance for coastal property and then taxes inland insurance companies to pay the difference between losses and premiums. Kelly and Kleffner (2003) show theoretically that disaster assistance leads households to decrease demand for insurance, causing a monopolistic insurer to lower the price. Tracy and Nickerson (1989) study adaptation decisions in a theoretical framework with disaster assistance, but no private insurance. Our results focus on how development subsidies and disaster assistance affect the tail of the damage distribution. We show that subsidies provide incentives to increase coastal property along the intensive and extensive margins, which increases the mass of the tail of the damage distribution, but does not affect the tail index.

Our results have important implications for the integrated assessment literature, which balances the costs of climate change abatement against the impacts of climate change, including cyclone damages. Integrated Assessment models that model cyclone damages use historical cyclone damage data to estimate damage parameters (e.g., Narita et al., 2009). Our results indicate that historical data is a poor predictor of future damages, due to the fat tail in the damage distribution. Indeed, Narita et al. (2009) note that “the coefficient is extremely sensitive to which period is chosen and averaged” and conduct extensive sensitivity analysis to address this issue.

Finally, our work touches on a large literature on adaptation. Much of the literature on adaptations (for example, Kelly et al., 2005; Mendelsohn et al., 1994) shows empirically how adaptations can reduce the point estimate of damages. Our complementary analysis provides theoretical insight into the process whereby adaptation policies affect the tail of the damage distribution.

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10 See Cavallo and Noy (2011) and Kousky (2014) for additional details on the state of the literature studying the economic costs of natural disasters.

11 In a recent paper, Jones (2015) creates a microfoundations model that generates a fat tail in the distribution of income. Further, in a theory in finance, fat tails arise in financial return data through random news events (for example, Peter and Clark, 1973; Haas and Pigorsch, 2011).

12 Gabaix (1999) proposes a theory for city sizes that generates a Pareto distribution. In their model, wages adjust to offset city-specific amenity shocks, which keeps the growth rate of cities independent of the size, implying a power law steady state distribution.

13 Sutter (2007) finds a 72% increase in population growth for coastal counties in states with insurance pools in the decade after implementation relative to the prior decade.
Data

The distributions of tropical cyclone damages, hurricane strength, and coastal population are explored using three data sets. Following the literature (e.g., Pielke et al., 2008; Pielke and Landsea, 1998), this study uses storm damage data from the ICAT insurance firm (http://www.icatdamageestimator.com) website, which provides damages from all tropical cyclones (hurricanes, tropical storms, and tropical depressions) making landfall in the United States between 1900 and 2012. The second data set is provided by a NASA website, which provides detailed storm-location and characteristic information (e.g., minimum pressure and maximum sustained wind speed) for all storms that originated in the Northern Atlantic Basin between 1850 and 2012. Given changes in satellite technology, only data since 1950 is used in the below analysis of storm strength. The third data set of coastal city populations is from the 2010 Census conducted by the US Census Bureau. Table 1 gives summary statistics for all data sets.

The ICAT data set aggregates data contained in the Monthly Weather Reviews, published since 1872, which provide summaries of the storms that have occurred in the North Atlantic Basin each year. These reports are currently published by employees of the National Hurricane Center, which is an office within the National Weather Service and the National Oceanic and Atmospheric Administration. The raw data includes storm name and the economic damages associated with the storm.

Economic damages are the direct losses from cyclone impact, assessed in the wake of the storm event. The damages do not include indirect or macroeconomic outcomes related to storm impacts, and include both insured and uninsured damages. Since 1987, the National Hurricane Center has calculated the total economic damages as twice the insured damages. This relationship is a guideline, not the result of a statistical estimation (Pielke et al., 2008, 1999). Several sources of uncertainty exist in the estimation of economic damages from natural disasters (Downton and Pielke, 2005; Downton et al., 2005). Previous consideration of these sources of uncertainty does not reveal any systematic bias in the calculation of storm damages from flooding events or tropical cyclones (Pielke et al., 2008; Downton and Pielke, 2005).

A number of features of coastal communities in the United States have changed through time in ways that might lead the economic damages caused by a storm to depend on the year in which the storm made landfall. Of principal importance is the amount of physical capital located in at-risk coastal communities. The total property in coastal communities has varied through time, though the value of this property is not historically recorded, confounding efforts to directly control for temporal variation in at-risk capital. Other related socioeconomic characteristics, such as per capita income and population are historically available. Using 2005 country-level data from the World Bank, Bakken and Mendelsohn (2016) find that capital scales proportionately with per capita income, estimating that capital stock is 2.65 times the per-capita income in a nation. In the absence of a time series of the capital stock of coastal communities in the United States between 1900 and 2012, we will rely on the population and income level of coastal communities to control for variation in capital stocks across time.

We adopt the methodology of Pielke et al. (2008), which controls for temporal changes in prices due to inflation and the population and wealth of coastal communities, by normalizing damages relative to 2012 prices, income, and population for each storm from 1900–2012 according to:

\[
D_{1,2012} = D_{1,y} \times I_{2012/y} \times RWPC_{2012/y} \times P_{2012/y}.
\]

Table 1

<table>
<thead>
<tr>
<th>Summary statistics.</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropical Cyclone Damages (billions of 2012 USD)</td>
<td>6.54</td>
<td>19.62</td>
<td>0.0001</td>
<td>183.32</td>
<td>241</td>
</tr>
<tr>
<td>Max. wind speed, Storms With Damage &gt; 0, 1900–2012 (MPH)</td>
<td>92.61</td>
<td>29.69</td>
<td>35</td>
<td>190</td>
<td>241</td>
</tr>
<tr>
<td>Max. wind speed, All Storms, 1950–2012 (MPH)</td>
<td>64.36</td>
<td>32.63</td>
<td>25</td>
<td>165</td>
<td>921</td>
</tr>
<tr>
<td>Storm Surge (Feet)</td>
<td>5.28</td>
<td>3.43</td>
<td>1</td>
<td>18</td>
<td>73</td>
</tr>
<tr>
<td>Storm Tide (Feet)</td>
<td>7.82</td>
<td>4.76</td>
<td>1</td>
<td>28</td>
<td>178</td>
</tr>
<tr>
<td>2010 Coastal City Population (1000s)</td>
<td>43.28</td>
<td>157.73</td>
<td>0.001</td>
<td>2504.7</td>
<td>3089</td>
</tr>
</tbody>
</table>
storms as measured by damages in 2012 dollars using the above methodology occurred prior to the end of World War II, when per capita income in coastal areas was much lower than modern levels.\textsuperscript{14}

Other time-varying characteristics of coastal communities are building codes and materials. Any improvements in building codes and materials over time tend to reduce the damages associated with more recent storms. Changes in building materials include reductions in the use of traditional building material such as wood in favor of reinforced concrete. Building codes in many coastal communities now specify preferred building footprints, materials, and techniques related to both energy efficiency and building stability.\textsuperscript{15} Many of these techniques, such as tighter nailing patterns for interior panels in high wind regions, increase the cost of construction relative to regions not at risk of storms. Higher construction costs are an additional cost of tropical cyclones that is not captured by the ICAT data set.

The distribution of storm strength will play an important role in the theoretical model and in the effects of adaptation, climate change abatement, and coastal development policies on aggregate damages. Therefore, we also use storm characteristic information on all cyclones formed in the Atlantic basin between 1950 and 2012, gathered from the Tropical Storm Tracks database maintained by the NASA Earth Science Data and Information System Global Hydrology Resource Center using data published by the National Hurricane Center (2013). The database provides location, category, minimum pressure, and maximum wind speed data for Atlantic basin storms from 1851.

The distribution of coastal property is also important for the theoretical model. We use coastal population data from the 2010 Census conducted by the US Census Bureau as a proxy for coastal property. The data gives population size within cities located in coastal counties. The term “city” refers to Census incorporated places, which can be cities, boroughs, towns, or villages, or other legally bounded entities depending on the state. The data set includes small towns (424 observations have less than 1000 people). Finally a data point exists for each county consisting of the balance of the county population living in unincorporated areas.

**Empirical methods**

The goal of the empirical analysis is to detect whether or not the distribution of tropical cyclone damages is fat tailed, which has implications for the efficacy of climate change abatement, adaptation, and development policies in coastal communities dealing with the threat of damage-inducing storms. Additional analyses explore the potential sources of the fat tail in the aggregate damages distribution.

The first step in the empirical methodology is the definition of fat tails. The terminology for different versions of fat tails varies in the literature. Here we say a distribution is *fat tailed* if it is asymptotically equivalent to a Pareto distribution (Roger and Nieboer, 2011). That is:

$$\lim_{D \to \infty} \frac{f(D)}{\alpha D^{-\alpha - 1}} \to 1, \quad \alpha > 0.$$

The above definition is particularly convenient because the *tail index* of a fat-tailed distribution is $\alpha$. The $k$th moment is infinite if and only if $\alpha < k$.

**Generalized pareto distribution**

The GPD is a popular choice for extreme value analysis, with applications in a variety of disciplines germane to the current effort, including: actuarial science (e.g., Brazouskas and Kleefeld, 2009; Cebrian et al., 2003), climatology (e.g., Nadarajah, 2008), and meteorology (e.g., Holmes and Moriarty, 1999). The distribution and density functions for the GPD, with shape parameter, $\xi$, and scale parameter, $\sigma$, are:

$$F_{\text{GPD}}(D; \sigma, \xi) = \begin{cases} 
1 - \left(1 + \frac{D}{\sigma}\right)^{-1/\xi} & \text{if } D > 0, \xi \neq 0 \\
1 - e^{-\frac{D}{\sigma}} & \text{if } \xi = 0 
\end{cases}$$

\textsuperscript{14} We also tried scaling by $RWPC^{1/2}$. An exponent less than one would occur if income growth is causing adaptations to increase at a rate greater than coastal property. The tail index is virtually unchanged, although the mean is lower, reflecting the assumption that adaptations increase at a greater rate over time.

\textsuperscript{15} Since 2000, the International Residential Code has been updated each three years with the latest recommendations regarding construction materials and techniques (International Code Council, 2003).
\[ f_{GPD}(D; \sigma, \xi) = \begin{cases} \left( \frac{1}{\sigma} \right) \left( 1 + \frac{D}{\sigma} \right)^{-\frac{1}{\xi}} \left( 1 - \frac{\xi}{D} \right) \left( 1 - \xi - \frac{\xi}{\sigma} \right)^{-1} & \text{if } D > 0, \xi \neq 0 \\ \left( \frac{1}{\sigma} \right) e^{-\frac{D}{\sigma}} & \text{if } \xi = 0 \end{cases} \] (4)

respectively. Note that \( 0 \leq D < \infty \) if \( \xi \geq 0 \), and \( 0 \leq D < -\frac{\sigma}{\xi} \) if \( \xi < 0 \).\(^{16}\)

The shape parameter affects the tail of the distribution as well as the existence of the moments for the distribution; the tail index is \( \alpha = 1/\xi \), so the \( k \) th moment exists if and only if \( \xi < \frac{1}{k} \) (Hosking and Wallis, 1987). The mean of the distribution, \( \sigma/(1 - \xi) \), is finite if and only if \( \xi < 1 \). When the mean is infinite, efforts to estimate distribution parameters based on maximized likelihood yield unreliable results. The variance, \( \sigma^2/(1 - \xi)^2(1 - 2\xi) \), is finite if and only if \( \xi < \frac{1}{2} \). Maximum likelihood estimation of GPD parameters is feasible when the distribution has infinite variance. For \( 0 < \xi \leq 0.5 \), the distribution decays at a rate slower than the normal distribution, but the variance is finite, and the central limit theorems still apply. When \( \xi < 0 \), the distribution is bounded above by \( \frac{\sigma}{\xi} \). When \( \xi = 0 \), the distribution is the exponential distribution. The area under the tail is monotonically increasing in \( \xi \). The probability of observing extreme values is greater than with a normal distribution if and only if \( \xi > 0 \). Therefore, the GPD nests thin (\( \xi \leq 0 \)) and fat (\( \xi > 0 \)) tails. Fig. 1 illustrates how the area under the tail changes as \( \xi \) takes on values between -1 and 1. Fig. 1 also gives an example with infinite variance (\( \xi = 2/3 \)) and infinite mean (\( \xi = 1 \)).

Our choice of the GPD distribution is appropriate for estimating the tail of the damage distribution for two reasons. First, a fairly general theorem (Pickands, 1975) shows that the tail of the GPD approximates the tail of any fat-tailed distribution (including Cauchy, Student-t, etc.). Further, the tail of heavy-tailed distributions with finite moments (such as log-normal) are asymptotically exponential, so the GPD covers this case as well. Second, since the GPD nests thin and fat tails, we can statistically test for thin versus fat tails using the estimate of \( \xi \).

Tests for fat tails

Our interest is in estimation of the shape of the tail of the distribution. Most of the literature uses subsets of the data that lie beyond threshold values (see de Zee Bermudez and Kotz, 2010, for a review of available techniques). Restricting the data set to values above a threshold is appropriate for two reasons. First, because extreme values are by definition rare, using the entire data set may result in an estimate of \( \xi \) that fits most of the empirical distribution well but is a poor fit for the tail (DuMouchel, 1983). Second, Pickands (1975) shows that a sufficiently large threshold exists such that the GPD approximates any distribution satisfying a technical condition above the threshold to an arbitrarily small approximation error. Therefore, the GPD is always appropriate for estimation of the tail, but may not be appropriate for estimation of the entire distribution.

\(^{16}\) Some versions of the GPD specify an additional location parameter, which corresponds to the lower bound of the support of the distribution. Here we follow the literature cited above and naturally set the location parameter equal to zero. The estimate of the shape parameter is not affected by the value of the location parameter (Pickands, 1975).
A tradeoff exists regarding the choice of threshold value: thresholds that are too high result in small sample sizes, which may result in small sample bias and high variance for the estimated parameters, while thresholds that are too low include ranges of the distribution for which the GPD may not be a good fit, leading to biased parameter estimates (Smith, 1987). Although a number of techniques exist, no clear rule exists for the selection of the threshold (Davison and Smith, 1990). Here we consider a variety of thresholds, and obtain similar results regardless of the threshold.

The analysis explores the presence of a fat tail in the tropical cyclone damage distribution using two methods. The first method directly estimates the shape parameter of the GPD via maximum likelihood. The second method uses weighted least squares (WLS) to estimate the slope of the mean-excess function, which is a function of the shape parameter (Ghosh, 2010). The tests utilize storm-level damage data from storms that made landfall between 1900 and 2012.\textsuperscript{16}

**Direct MLE estimation**

Consistent and asymptotically normal maximum likelihood estimators (MLEs) of the shape and scale parameters exist for \(-1/2 < \xi < 1\). Giles et al. (2016) derives analytic expressions for the small-sample bias of MLE of the parameters of the GPD. We also reported bias-corrected parameter values.

**Mean-excess estimation**

The distinguishing feature of a fat-tailed distribution, relative to a thin-tailed distribution, is that the expected size of a draw larger than any draw yet observed is much larger than the largest draw to date. Given a threshold \(u\), the mean-excess function is:

\[
M(u) \equiv E[X - u | X > u],
\]

as long as \(E[X_i] < \infty\). A distribution is fat tailed if \(M(u)\) is increasing in \(u\). The implication is that previous realizations from a fat-tailed distribution are poor predictors of the magnitude of observations from the tails of the distribution.\textsuperscript{19} Distributions can be identified as fat tailed through the mean-excess function.

The empirical estimate of the mean-excess is:

\[
\hat{M}(u) = \frac{\sum_{i=1}^{n} (X_i - u)I[X_i > u]}{\sum_{i=1}^{n} I[X_i > u]}
\]

The slope of the mean-excess function can be used to estimate \(\xi\), as the mean-excess function of a GPD is linear in \(u\) (Ghosh, 2010):

\[
M(u) = \frac{\sigma}{1 - \xi} + \frac{\xi}{1 - \xi}u.
\]

The slope of the mean-excess function for a GPD random variable is positive if the GPD has fat tails and finite expected value (0 < \(\xi\) < 1). Fig. 2 illustrates the mean-excess function for the GPD when \(\xi = 2/3\).

We use WLS to estimate the mean-excess function (7). The data set consists of a set of minimum and maximum thresholds and empirical mean excess values computed via equation (6). The maximum threshold must not be too large, as data points with too few terms result in the average (6) being a noisy estimate of the true mean excess (Ghosh, 2010).

The number of data points greater than the threshold is monotonically decreasing in the threshold, so the mean excess of smaller thresholds are more precisely estimated. The standard approach using WLS is to assign weights based on the precision of the estimated mean-excess. Then the weights monotonically decrease as the threshold increases, which means that WLS puts little emphasis on fitting the tail. Therefore, we use two weighting schemes to deal with the heteroskedasticity in the mean excess data. The first weights by the number of observations used to calculate the mean excess, while the second weights by the inverse of the number of observations.

**Empirical results**

**Fat-tailed damages**

Table 2 reports estimates of the shape parameter for storm level damage data, during the period 1900–2012, using MLE.

\textsuperscript{17} In a sense, the empirical dilemma of choosing a threshold mirrors the difficulty of pricing insurance when the damage distribution may be fat tailed: a dearth of extreme values and sensitivity of the shape parameter to the choice of threshold makes estimating the probability of extreme events difficult.

\textsuperscript{18} Given that some insurance contracts are annual, MLE of annual damages may be theoretically more relevant than data at the individual storm level. However, evidence exists that prospective homeowners may make choices about property purchases and insurance coverage in response to the damages caused by a single storm (Gallagher, 2014; Bin and Polasky, 2004; Beron et al., 1997).

\textsuperscript{19} In contrast, new records in, for example, athletic events tend to be close to old records, indicating that results from athletic events are thin-tailed.
Given the lack of guidance about the appropriate threshold for use in estimation of the shape parameter, Table 2 reports MLEs of $x$ for several thresholds. Nonetheless, a thin-tailed distribution ($\xi \leq 0$) is rejected at the 95% confidence level for all threshold choices. Further, all point estimates are in the region indicating a fat-tailed distribution with finite mean but infinite variance, $0.5 < \xi < 1$. Of course, the total property at risk is finite and thus the variance of the distribution cannot truly be infinite. Instead, the interpretation of the results is that the total property at risk is large enough so that the data cannot statistically distinguish between a distribution with a large truncation point and thus finite variance, and an unbounded distribution with infinite variance. Indeed, the total value of insured property in FL alone is $3.6$ trillion (FPHLM, 2016), a far greater amount than the damages caused by the most damaging hurricane.

For lower thresholds, more data is available, increasing the precision of the estimates. However, the additional data implies the shape parameter is chosen to fit a wider range of the tail of the damage distribution. In fact, a larger shape parameter fits the middle part of the damage distribution best. In contrast, a smaller shape parameter fits the rightmost part of the tail of the damage distribution best. Restricting the data set to larger thresholds comes at a price of reducing the number of observations. Therefore, the standard errors widen, and finite variance cannot be rejected at the 95% confidence level for thresholds equal to the 60th or 80th percentiles. Nonetheless, the point estimates continue to indicate a fat tail with finite mean but finite variance, even at the 80th percentile.

Fig. 3 illustrates the threshold problem. The figure shows a histogram of the data, along with the GPD distribution with parameters determined using MLE on the entire data set as well as on only the data greater than the median ($4.55$ billion). The histogram has a long right tail, with several observations greater than $50$ billion. From the figure, using the entire data set fits the data best for damages less than the median. In contrast, for damages greater than the median (the tail), the threshold estimation has a better fit. For example, using the whole data set results in estimates which severely underestimate the probability of damages between $20$ and $30$ billion.

Table 2 also reports MLEs corrected for small sample bias using the results of Giles et al. (2016). The bias-correction procedure increases the coefficients, but all point estimates remain in the fat-tailed region with finite mean and infinite variance. The bias-corrected coefficients are also more similar across thresholds. The results in Table 3 provide even stronger support for the conclusion that the distribution of tropical cyclone damages is fat tailed. When the upper threshold is chosen at the 70th or 75th percentile, so that each mean excess is calculated using at least 49 observations, the point estimates of $\xi$ lie in a tight range, from $0.69$ to $0.74$. These point estimates indicate the distribution is fat tailed, with a finite mean and infinite variance. Further, the bootstrapped 95% confidence intervals are all between 0.5 and 1, indicating a thin tail, infinite mean, and finite variance are all rejected at the 95% confidence level.

Table 3 also reports results generated assuming the upper threshold equals the second largest data point. For the second largest data point, the largest mean excess is estimated using a single observation and the larger mean excess data points are estimated with very few data points. The resulting additional noise in the mean excess estimates increases the standard

---

20 We do not report the estimates of the scale parameter, $\hat{\sigma}$, as the tail index is our primary focus. Although thresholds that result in higher $\hat{\xi}$ estimates often also have a lower estimate of $\hat{\sigma}$, estimated mean damages, $\hat{\sigma}/(1 - \hat{\xi})$, increase with $\hat{\xi}$ for all but one threshold.
errors by almost a factor of 10 and causes the point estimates to vary across specifications. Nonetheless, a thin tail is rejected at the 95% confidence level for both specifications. However, both point estimates indicate that the variance is finite when the upper threshold equals the second largest data point.

Of primary interest to insurance firms and regulators is the damage from a catastrophic storm. The damage from a catastrophic storm is important for public policies such as spending on adaptation infrastructure and building code regulations (Kunreuther et al., 2004). Further, the damage from a catastrophic storm is of primary interest for regulators setting minimum reserve requirements for the insurance industry (Kousky and Cooke, 2012). Let \( D^* \) denote the minimum damage from the \( 1 - 1/T \) upper quantile of the damage distribution. We then say a storm is catastrophic if the storm results in damage

### Table 2
MLE of the shape parameter for the storm damage distribution. The bias adjusted \( \tilde{\xi} \) is the value of \( \xi \) after making the small sample bias adjustment shown in Giles et al. (2016).

<table>
<thead>
<tr>
<th>Threshold</th>
<th>( \tilde{\xi} )</th>
<th>Confidence interval</th>
<th>Bias adjusted ( \tilde{\xi} )</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>45th percentile</td>
<td>0.946</td>
<td>0.593–1.299</td>
<td>0.992</td>
<td>133</td>
</tr>
<tr>
<td>Median</td>
<td>0.873</td>
<td>0.52–1.227</td>
<td>0.925</td>
<td>119</td>
</tr>
<tr>
<td>55th percentile</td>
<td>0.902</td>
<td>0.512–1.292</td>
<td>0.959</td>
<td>108</td>
</tr>
<tr>
<td>60th percentile</td>
<td>0.806</td>
<td>0.42–1.192</td>
<td>0.873</td>
<td>95</td>
</tr>
<tr>
<td>65th percentile</td>
<td>0.751</td>
<td>0.356–1.146</td>
<td>0.829</td>
<td>84</td>
</tr>
<tr>
<td>70th percentile</td>
<td>0.668</td>
<td>0.274–1.061</td>
<td>0.762</td>
<td>72</td>
</tr>
<tr>
<td>75th percentile</td>
<td>0.764</td>
<td>0.271–1.258</td>
<td>0.872</td>
<td>60</td>
</tr>
<tr>
<td>80th percentile</td>
<td>0.618</td>
<td>0.141–1.095</td>
<td>0.763</td>
<td>48</td>
</tr>
</tbody>
</table>

### Table 3
WLS estimation of the shape parameter using the mean-excess function for the storm damage distribution. The confidence intervals were generated by bootstrapping. Regressions with higher thresholds have more observations, but include larger observations where the mean excess data is less precise. In the second column, “none” indicates the maximum possible number of observations, removing only the largest data point. The third column indicates that weighting is done either by the number of data points used to calculate the mean excess, or the inverse.

<table>
<thead>
<tr>
<th>Lower threshold</th>
<th>Upper threshold</th>
<th>Weighting</th>
<th>( \tilde{\xi} )</th>
<th>Std. err.</th>
<th>Confidence interval</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>30th percentile</td>
<td>70th percentile</td>
<td>Obs.</td>
<td>0.73</td>
<td>0.011</td>
<td>0.71–0.75</td>
<td>79</td>
</tr>
<tr>
<td>30th percentile</td>
<td>70th percentile</td>
<td>Inverse obs.</td>
<td>0.71</td>
<td>0.011</td>
<td>0.69–0.73</td>
<td>79</td>
</tr>
<tr>
<td>35th percentile</td>
<td>70th percentile</td>
<td>Obs.</td>
<td>0.70</td>
<td>0.010</td>
<td>0.68–0.72</td>
<td>70</td>
</tr>
<tr>
<td>35th percentile</td>
<td>70th percentile</td>
<td>Inverse obs.</td>
<td>0.70</td>
<td>0.010</td>
<td>0.68–0.72</td>
<td>70</td>
</tr>
<tr>
<td>30th percentile</td>
<td>75th percentile</td>
<td>Obs.</td>
<td>0.72</td>
<td>0.008</td>
<td>0.70–0.73</td>
<td>88</td>
</tr>
<tr>
<td>30th percentile</td>
<td>75th percentile</td>
<td>Inverse obs.</td>
<td>0.70</td>
<td>0.009</td>
<td>0.68–0.71</td>
<td>88</td>
</tr>
<tr>
<td>35th percentile</td>
<td>75th percentile</td>
<td>Obs.</td>
<td>0.69</td>
<td>0.008</td>
<td>0.68–0.71</td>
<td>79</td>
</tr>
<tr>
<td>35th percentile</td>
<td>75th percentile</td>
<td>Inverse obs.</td>
<td>0.69</td>
<td>0.009</td>
<td>0.68–0.72</td>
<td>79</td>
</tr>
<tr>
<td>35th percentile</td>
<td>none</td>
<td>Obs.</td>
<td>0.24</td>
<td>0.09</td>
<td>0.06–0.40</td>
<td>126</td>
</tr>
<tr>
<td>35th percentile</td>
<td>none</td>
<td>Inverse obs.</td>
<td>0.43</td>
<td>0.054</td>
<td>0.33–0.54</td>
<td>126</td>
</tr>
</tbody>
</table>

**Fig. 3.** Histogram of the damage data after adjusting to 2012 income, population, and prices and maximum likelihood estimation fit using the GPD.
large enough so that such storms occur with probability $1/T$. For example, if $n_s$ storms occurred per year and $T = 100n_s$, then a catastrophic storm would be a once-in-a-hundred-years storm. Higher values of $D^*$ imply that a catastrophic storm is more damaging. From the definition of a catastrophic storm:

$$\text{Prob}(D \geq D^*) = \frac{1}{T},$$

(8)

$$\text{Prob}(D \leq D^*) = 1 - \frac{1}{T}.$$  

(9)

$$D^* = F^{-1}_{\text{GPD}} \left( 1 - \frac{1}{T}, \sigma, \xi \right).$$

(10)

Eq. (10) adjusts $D^*$ so that the mass of the tail is one percent for $T = 100n_s$. If the mass of the tail increases, then $D^*$ increases until the mass of the distribution to the right of $D^*$ is again one percent. Thus, a higher value of $D^*$ indicates the tail has more mass. Finally, equation (10) shows that $D^*$ and the mass of the tail depend on both $\sigma$ and $\xi$.

Table 4 gives the estimates of the damage from a once-in-a-hundred-years storm using the GPD parameters estimated in Table 3 and given a mean of 2.73 storms per year over the past 50 years. The point estimates range from about $166-$214 billion. Hurricane Katrina caused the maximum damage in only the last 50 years, at approximately $96$ billion. Looking at data in the last 100 years, the most damaging hurricane was the Great Miami Hurricane in 1926 with an estimated $183$ billion in damages.22 Table 4 also gives the damage for a once-in-a-hundred-years storm based on parameters estimated assuming damages are normally distributed and log normally distributed. The damage estimate for a once-in-a-hundred-years storm is only $31$ billion for the thin-tailed normal distribution, which is apparently a poor estimate given that 4 storms exceeded $35$ billion of damages in only the last 50 years. The log normal distribution has a much higher estimate ($448$ billion), largely because the whole data set is used, and so the parameters of the distribution fit the middle of the distribution well, but have a poor fit in the tail.

These results illustrate the importance of welfare of properly accounting for a fat tail in the damage distribution. Assuming a fat-tailed distribution increases the tail risk by a factor of about 5.7 relative to a thin-tailed distribution. To isolate the effect of tail risk on welfare, suppose the increase in tail risk was mean-preserving. A welfare cost of the fat tail then arises because insurance firms must increase their costly reserves. If, for example, insurance firms hold enough reserves to cover an annualized one percent value at risk, then the fat tail means insurance firms must hold an additional $135-$183 billion in reserves relative to the thin-tailed normal distribution. These costs are passed on to households in terms of higher prices. Households then purchase less insurance, which increases household risk and reduces the risk pool. The welfare cost of a mean-preserving increase in tail risk is then the increase in risk faced by uninsured households.

An interesting exercise is to compare our results to other estimates of cyclone damage tail risk. While insurance firms generally use proprietary risk models, many publicly available models either assume a thin tail or a log normal distribution not specifically designed to fit the tail of the distribution.23 For example, the widely used Florida Public Hurricane Loss Model (FPHLM) conducts sophisticated Monte Carlo simulations of “bottoms up” engineering and wind speed models to obtain a distribution for insured damages, assuming the underlying random variables are thin tailed. FPHLM estimates the insured

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21 The minimum damage $D^*$ can also be interpreted as the annualized value at risk using probability $n_s/T$.
22 Note also that Florida alone has $3.6$ trillion in insured properties (FPHLM, 2016).
damage from a once-in-a-hundred-years storm at $59 billion (FPHLM, 2016). This corresponds to about $118 billion including non-insured damages, which is within our 95% confidence interval, but less than our point estimate. Note finally that although it is common to compare statistical and Monte Carlo models, the modeling approaches are quite different. In addition, FPHLM assesses vulnerability in Florida only, whereas our data is nationwide. Other states may have more structural vulnerabilities (for example, weaker buildings).

While many insurance firms and regulators are no doubt aware that a fat tail exist, the results of Table 4 shows that estimates of tail risk assuming a fat tail are large and have wide confidence intervals. This motivates exploration of the cause of the fat tail, which we turn to next.

**Distribution of wind speed**

We next perform empirical tests for fat tails identical to the Fat-tailed damages section, but using data on storm strength, measured by maximum wind speed, rather than damages. Air friction, limitations on the difference between atmospheric and sea surface temperatures, and other factors imply theoretical bounds on tropical cyclone intensity (Emanuel, 1995). The mean wind speed conditional on the speed exceeding a threshold becomes very close to the threshold as threshold approaches the theoretical bound. Therefore, the wind speed distribution is likely to be thin tailed. Showing that the wind speed distribution is indeed thin tailed eliminates one possible cause of fat tailed damages in the Theoretical model section.

Table 5 reports the results from WLS estimation of the mean excess (the MLE results are similar). Table 5 shows that, regardless of the lower and upper thresholds, the point estimate of the tail index is always negative, indicating that the wind speed distribution is thin tailed. In all specifications, a fat tail is rejected at the 95% confidence level.

**Coastal population distribution**

Another possible cause of fat-tailed damages of the Theoretical model section is a fat tail in the property distribution. Table 6 reports the results of WLS estimation of the coastal population data (our proxy for the property distribution). Table 6 reports shape parameter estimates between 0.75 and 0.78, again rejecting a thin tail at the 95% confidence level. Further, the shape parameter estimates are very close to the shape parameter estimates for the aggregate damages. In the Theoretical results, we show that the theoretical model predicts that the fat tail in the damage distribution arises from a fat tail in the property distribution and that the shape parameter estimates of damages and the property distributions are equal.

Although the coastal city population and property levels are correlated, the use of population as a proxy introduces two potential concerns. First, the geographic area of each city is not constant. Vulnerability to catastrophic cyclones may differ for cities with geographically concentrated populations as opposed to cities which are less geographically dense. Second, the data lumps all the unincorporated population in a county into a single data point. One way to address the geographic area issue is to use the G-Econ data set from Nordhaus et al. (2006). This data set divides coastal counties into a grid and computes the population in the grid. A disadvantage of this data set is that the grid is relatively coarse, with only 53 observations. Mean excess estimation of the shape parameter of the GPD using the G-Econ data set also rejects a thin tail at the 95% confidence level. Further, the shape parameter estimates between 0.75 and 0.78, again rejecting a thin tail at the 95% confidence level for all specifications (results available from the authors upon request). The point estimates are slightly lower, between 0.40 and 0.55. To address potential issues with the unincorporated areas, we also compare results including/excluding the unincorporated areas. Excluding unincorporated areas slightly reduces the point estimates to 0.66–0.71, depending on the thresholds. A thin tail and finite variance are rejected for all specifications.

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24 FPHLM simulates wind speeds in a particular zip code, then estimates damage to properties in the zip code using information on building types, insurance coverage, and other factors. The path of the storm is one important factor in determining the damage from a catastrophic storm.

25 Maximum wind speed is a commonly used measure of the strength of storms (see, for example Nordhaus, 2010). Other measures of storm strength, such as size, are correlated with wind speed. Further, we find that the distributions of storm surge and storm tides are all thin tailed using the mean-excess methodology on a Gulf of Mexico storm surge data set (Needham and Keim, 2012). Details are available on request.
Table 6
WLS estimation of the shape parameter using the mean-excess function for the coastal city population distribution.

<table>
<thead>
<tr>
<th>Lower threshold percentile</th>
<th>Upper threshold percentile</th>
<th>Weighting</th>
<th>( \hat{\tau} )</th>
<th>Std. err.</th>
<th>Confidence interval</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>35th</td>
<td>70th</td>
<td>Obs.</td>
<td>0.781</td>
<td>0.0003</td>
<td>0.781–0.782</td>
<td>1092</td>
</tr>
<tr>
<td>35th</td>
<td>70th</td>
<td>Inverse Obs.</td>
<td>0.782</td>
<td>0.0003</td>
<td>0.781–0.782</td>
<td>1092</td>
</tr>
<tr>
<td>40th</td>
<td>70th</td>
<td>Obs.</td>
<td>0.784</td>
<td>0.0006</td>
<td>0.783–0.786</td>
<td>936</td>
</tr>
<tr>
<td>40th</td>
<td>70th</td>
<td>Inverse Obs.</td>
<td>0.782</td>
<td>0.0007</td>
<td>0.781–0.783</td>
<td>936</td>
</tr>
<tr>
<td>35th</td>
<td>90th</td>
<td>Obs.</td>
<td>0.751</td>
<td>0.0007</td>
<td>0.75–0.753</td>
<td>1713</td>
</tr>
<tr>
<td>35th</td>
<td>90th</td>
<td>Inverse Obs.</td>
<td>0.751</td>
<td>0.0009</td>
<td>0.748–0.752</td>
<td>1713</td>
</tr>
<tr>
<td>40th</td>
<td>90th</td>
<td>Obs.</td>
<td>0.747</td>
<td>0.0007</td>
<td>0.745–0.748</td>
<td>1557</td>
</tr>
<tr>
<td>40th</td>
<td>90th</td>
<td>Inverse Obs.</td>
<td>0.747</td>
<td>0.0009</td>
<td>0.745–0.749</td>
<td>1557</td>
</tr>
</tbody>
</table>

Theoretical model

We consider a theoretical model of aggregate damages, which consists of three major parts. The first part consists of a model of household behavior. Households decide whether or not to live on the coast and how much insurance and property to purchase. In the second part of the model, households living on the coast organize into population centers. Finally, a model of storm strength relative to the strength of adaptations determines the probability that a population center is in the path of a storm. Fig. 4 illustrates the three parts of the model.

Household behavior

A continuum of households with measure one have preferences for consumption, \( c \), and property. “Property” here refers to the structures and personal property on the land, not the land itself. We denote the quantity of property for household \( i \in [0,1] \) as \( A_{ij} \), where \( j \in \{K,I\} \) denotes the property type (coastal, \( K \), or inland, \( I \)). One can think of property \( A_{ij} \) as the dollar value of all items owned by the household on the land. Households have heterogeneous preferences for property. The period utility function for household \( i \) is:

\[
U_i = u(c_i) + v(\phi(i)A_{K,i} + A_{I,i}).
\]  

Eq. (11) assumes property types are perfect substitutes, which implies households will either buy coastal or inland properties, but not both. Property is homogeneous except for type. Without loss of generality, we order households from least to strongest preference for the coastal property, so that \( \phi(i) \) is increasing. We assume \( \phi(0) = 0 \) and \( 2q_r \leq \phi(1) < \infty \), where \( q_r = q_K/q_I \) is the price ratio (coastal over inland property). The assumptions on \( \phi \) ensure that the household with the smallest preference for coastal property lives inland and the household with the strongest preference lives on the coast. We further assume the Inada conditions hold for \( u \) and \( v \), so that total property (coastal plus inland) is positive for each household.

Hurricane damage and insurance

Only the coastal property is at risk of damage from tropical cyclones. Let \( h \) be a random variable that equals one if a unit of coastal property suffers a loss from a storm and that equals zero otherwise. Conditional on \( h = 1 \), the fractional loss of property the household suffers is \( d_i(h) \). We assume that \( d \) and \( h \) are independent. Presumably both the probability of damage and the amount of damage increase with the strength of the storm. Positive correlation between \( d \) and \( h \) would strengthen the results, but at a cost of considerable complexity.

Let \( q_K \) and \( q_I \) denote the price per unit of coastal and inland properties, respectively. We assume both types of property can be produced at constant marginal cost. The cost of coastal property includes any adaptation costs such as the cost of compliance with stricter building codes. Perfect competition among suppliers of property implies \( q_K = MC_K \) and \( q_I = MC_I \).

The total value of assets at risk is therefore \( q_K \int_0^1 A_{K,i}dh = q_KA_K=Nq_Kn \). It is convenient to treat each unit of property (as opposed to each individual) as potentially suffering a loss. That is, some units of an individual’s property may suffer a loss, while other units do not. Let \( N \in [0,1,\ldots,n] \) be the (random) total units of property which suffer any loss. If \( N \) has a continuous distribution, then for any realization, \( N \), we take the highest integer less than or equal to \( N \). Total losses are then:

\[
D = q_K \sum_{i=1}^{N} d_i. 
\]  

According to Eq. (12), each of \( N \) units of coastal property experience a loss, and the losses are realizations \( d_i \). One could equivalently (given independence) sum over households and let the distribution of \( d \) grow as households acquire more property. However, the above formulation simplifies the limiting arguments presented later.

\[26\] Adding a market for land would not substantively change the results.
Households may purchase an insurance contract which reimburses the household for property lost from a storm. Let $p(x)$ be the insurance premium per unit of insured property, where $x$ is the co-insurance rate. A large number of risk neutral firms provide insurance in a competitive market. Total expected profits equal premiums less expected losses:

$$E[\pi] = p(x)n - xE[D].$$

(13)

Applying Wald’s lemma to (12) gives:

$$E[\pi] = p(x)n - xqK\hat{N}E[d] = p(x)n - xqK\hat{N}\hat{d}.$$  

(14)

Here $\hat{d}$ is the expected fraction of loss a damaged property receives. Eq. (14) assumes that the expected fraction of total properties which suffer damage ($\hat{N}$) is independent of $n$.28

Perfect competition implies profits are zero, in which case the premium equals the expected reimbursement per unit of property:

$$p(x) = xqK\hat{N}\hat{d}.$$  

(15)

Government

A government agency exists that reimburses all coastal property owners a fraction $\tau$ of their losses. The government agency is funded by a lump sum tax on all households (irrespective of whether the households own coastal property). We assume a large population so that tax payments are a negligible fraction of total income. The reimbursement of losses can be thought of as a disaster relief agency, such as FEMA. FEMA reimburses households only for uninsured losses up to a limit of $33,000 in 2016, and only for certain types of property (FEMA, 2008). To model these limits in a parsimonious way, the model assumes $\tau$ fraction of property is of the type FEMA reimburses.29

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27 Homeowner’s insurance reimburses other losses, such as temporary housing costs. Conversely, some losses such as lost time are not insurable. For simplicity, we focus only on property, which is consistent with the empirical damage distribution.

28 That is, adding more property does not change the probability that another existing property is damaged by a storm. If $\hat{N}$ was a decreasing function of $n$, then property insurance becomes less expensive as additional properties are added. This creates a feedback mechanism, where disaster relief encourages the purchase of additional coastal property, which decreases the cost of insurance, which encourages still further coastal property building. Such a feedback mechanism is not likely to be large, but adds considerable complexity to the problem.

29 Following Kelly and Kleffner (2003), we assume $\tau$ is exogenous. In reality, the likelihood of a FEMA disaster declaration and associated spending is positively related to the size of the damage among other factors. If $\tau$ was endogenous, our results would strengthen. Households in areas with more property at risk would have greater incentives to accumulate property, knowing that a disaster declaration is likely. The property distribution would then become more skewed.
The subsidy is also consistent with other actual policies. The subsidy is equivalent (in the sense of producing an identical incentive to over-accumulate coastal property) to direct government provision of insurance at a subsidized price $p(x) - \tau E[D]$. Examples of government provision of property insurance include the National Flood Insurance Program (NFIP) and Florida’s Citizen’s Property Insurance. Jenkins (2005) estimates some NFIP premiums are 35–40% below actuarially fair levels. Finally, our subsidy is also equivalent to “wind/beach pools” whereby the government offers subsidized insurance for coastal property, and then taxes inland insurance companies to pay the difference between losses and premiums (Sutter, 2007). Indeed, given the limit on reimbursement by FEMA, the over-accumulation of property caused by $\tau$ is probably best interpreted as the result of subsidized insurance.

In addition, the government subsidizes the cost of coastal property at rate $\lambda$ per dollar. Examples include coastal development projects undertaken by the Army Corps of Engineers or infrastructure projects paid for by state governments (see Bagstada et al., 2007, for a discussion of coastal development subsidies).

The government budget constraint then sets tax revenue, $T$, equal to total reimbursement expenditures, $\tau D$, plus total coastal property subsidies, $\lambda q_k A_K$:

$$T = \tau D + \lambda q_k A_K = \tau q_k \sum_{j=1}^{N} d_j + \lambda q_k A_K. \quad (16)$$

Total reimbursement is a random variable, so Eq. (16) implies tax payments are random. In practice, FEMA keeps a reserve fund to smooth taxes over time. Further, the government can borrow (and has in fact borrowed) to smooth tax payments in the event the fund is exhausted. Here we assume FEMA can perfectly smooth tax payments over time. Therefore, taxes equal the mean of total disaster reimbursements plus total coastal property subsidies:

$$T = \tau E[D] + \lambda q_k A_K = \tau \eta q_k \bar{h} d + \lambda q_k A_K. \quad (17)$$

Household problem

The budget constraint sets household endowment income, $\omega$, plus disaster relief equal to expenses, which are consumption, the tax, the uninsured storm losses (losses less insurance payouts), the insurance premium, and the purchase of property.

$$\omega + \tau q_k \sum_{j=1}^{A_{K_j}} h_j d_j = c_t + T + (1 - x) q_k \sum_{j=1}^{A_{K_j}} h_j d_j + p(x) A_{K,i} + q_k (1 - \lambda) A_{K,i} + q_t A_{t}, \quad (18)$$

Note that the property subsidy applies only to the initial purchase, not the repair cost paid by either the insurance company or the household after the damage. The timing of the problem is such that at the beginning of the period the household purchases insurance and enjoys property. Then losses are realized. The household must spend endowment income replacing any damaged property not already reimbursed by insurance or disaster relief. At the end of the period, the household consumes the remaining endowment.

Substituting the actuarially fair insurance premium (15) into the budget constraint and the budget constraint into the utility function (11) results in the household problem:

$$\max E[U] = \max_{x, A_{K,i}, A_{t}} \left[ u \left( \omega - T + (\tau - (1 - x)) q_k \sum_{j=1}^{A_{K,i}} h_j d_j - q_k (1 - \lambda) A_{K,i} - q_t A_{t} \right) \right] + v \left[ \phi(i) A_{K,i} + A_{t} \right]. \quad (19)$$

In problem (19), each unit of coastal property is subject to an iid distributed loss. Hence the households take expectations with respect to each unit of property at risk. We model the co-insurance rate as a control variable. Rules vary in practice, but in many states lenders can only mandate that households insure the value of structures. Thus, households have discretion as to whether or not to insure personal property and households without a mortgage have discretion over insuring structures as well. Problem (19) implies the disaster relief agency effectively subsidizes the acquisition of coastal property.

---

30 None of the results would change if inland development was also subsidized.
31 The model allows for subsidizing repair costs by appropriately modifying $\tau$.
32 In practice, a household might elect not to replace all damaged property.
An equilibrium given a government policy $\tau$ is a set of household decisions $\{x, A_{K,j}, A_{I,j}\}$, prices $[p,q_k,q_I]$, and a government tax rate $T$ such that households and firms optimize, the government budget constraint holds, and supply equals demand in the property markets: $q_K = MC_K$ and $q_I = MC_I$.

**Results: Household problem**

We first derive a set of preliminary results for the household problem. The government insures part of the risk through the disaster relief agency, and we show that the household buys enough insurance to remove any remaining risk. The first order condition for the optimal co-insurance rate is:

$$E_{h_j} u' \left( \omega - T + (\tau - (1 - x))q_k \sum_{j=1}^{A_{K,j}} h_j d_j - xq_k \bar{h} d_k A_{K,j} - q_k (1 - \lambda) A_{K,j} - q_k A_{I,j} \right) = 0. \quad (20)$$

We hypothesize that the household insures all risk not covered by the government agency. That is, we hypothesize that $x = 1 - \tau$ satisfies the first order condition (20), making it the optimal co-insurance rate. Substituting the proposed solution into the first order condition results in:

$$u' \left( \omega - T - q_k (1 - \tau) A_{K,j} \bar{h} d_k - q_k (1 - \lambda) A_{K,j} - q_k A_{I,j} \right) \cdot E_{h_j} \left[ \sum_{j=1}^{A_{K,j}} h_j d_j - \bar{h} d_k A_{K,j} \right] = 0. \quad (21)$$

because marginal utility is not stochastic under the proposed solution. Further, $h_j$ and $d_j$ are iid, so Eq. (21) simplifies to:

$$\sum_{j=1}^{A_{K,j}} E[h_j] E[d_j] = \bar{h} d_k A_{K,j}. \quad (22)$$

Eq. (22) holds, which verifies that the assumed behavior is optimal. Therefore, the household insures all risk not covered by the disaster relief agency. That disaster relief causes households to reduce demand for insurance is known as a “charity hazard.”

Our results indicate that households insure all risk that is not covered by disaster relief. Alternative reasons for a lack of full insurance include search costs, underestimated risk by households, and insurance priced above actuarially fair levels due to reserve costs (Kousky and Cooke, 2012). These reasons matter only if demand for insurance falls below $1 - \tau$ and the household takes on risk. If households incur some risk, then the effects on demand for property depend on the reason households are not fully insured. For example, households that underestimate risks may purchase more property than in our model, underestimating the probability of loss. Conversely, if households under-insure because the price of insurance is above the actuarially fair price, then households would buy less property, to reduce uninsured risk.

We next derive the purchase of coastal versus inland properties. Substituting the optimal insurance rate $x = 1 - \tau$ into the problem (19) gives:

$$\max_{A_{K,j}, A_{I,j}} u \left[ \omega - T - (1 - \tau) \bar{h} d_k A_{K,j} - q_k (1 - \lambda) A_{K,j} - q_k A_{I,j} \right] + v [\phi(i)A_{K,j} + A_{I,j}] \quad (23)$$

The next proposition shows that a cutoff household exists such that all households with stronger preferences for coastal property than the cutoff household will buy only coastal property. All other households will buy only inland property.

**Proposition 1.** Assume the Inada conditions hold for $u$ and $v$, $\phi(0) = 0$, $2q_r < \phi(1) < \infty$, and $\phi$ is continuous and increasing. Then, there exists a cutoff household $i^* \in (0, 1)$ such that:

$$\phi(i^*) = q_r \left( 1 - \lambda + (1 - \tau) \bar{h} d_k \right). \quad (24)$$

Further,

1. $A_{I,j} = 0$ for all $i \in (i^*, 1)$.

---

33 The result that households do not fully insure in the presence of disaster relief is derived assuming competitive insurance markets, which implies insurance is actuarially fair. Introducing market power in the insurance market would exacerbate this result in an intuitive way, while adding significant complexity to the derivation of later results.

34 Although common in theoretical and policy discussions (e.g. Bagstadia et al., 2007), empirical evidence for the charity hazard is mixed (Raschky et al., 2013).
2. \( A_{K,i} = 0 \) for all \( i \in [0, i^*] \).

Given that each household holds only one type of property, the first-order conditions become:

\[
u'[\omega - T - q_k \left( 1 - \lambda + (1 - \tau) \bar{h}d \right) A_{K,i}] q_k \left( 1 - \lambda + (1 - \tau) \bar{h}d \right) = \phi(i) v'[\phi(i) A_{K,i}], i > i^*, \tag{25}\]

\[
u'[\omega - T - q_i A_{ij}] q_i = v'[A_{ij}], i < i^*. \tag{26}\]

Eq. (25) shows that demand for coastal property is determined by the effective price of coastal property \( q_k(1 - \lambda + (1 - \tau) \bar{h}d) \), which equals the subsidized cost of purchase, plus the subsidized cost of insuring the property.

For \( i < i^* \), the first-order condition is independent of \( i \). Therefore, households purchase identical amounts of inland property: \( A_j = i^* A_{ij} \). The demand for coastal property may vary by household. General comparative statics and analytical solutions for special cases are available for Eq. (25):

**Proposition 2.** Let the assumptions of Proposition 1 hold. Then:

**Proposition 2.1.** The fraction of households living on the coast, \( 1 - i^* \), is increasing in \( \tau \) and \( \lambda \).

**Proposition 2.2.** The quantity of coastal property, \( A_{K,i} \), is increasing in \( \tau \) and \( \lambda \) for all \( i \in [i^*, 1] \).

**Proposition 2.3.** The total replacement cost of coastal property at risk, \( q_k n = q_k \int A_k dt \), is increasing in \( \tau \) and \( \lambda \).

**Proposition 2.4.** For the special case of \( U_i = \log C_i + \log(\phi(i) A_{K,i} + A_{ij}) \), total property at risk is:

\[
n = \frac{(w - T)(1 - i^*)}{2q_k \left( 1 - \lambda + (1 - \tau) \bar{h}d \right)}. \tag{27}\]

**Proposition 2.5.** Per household purchase of inland property, \( A_{ij} \), is independent of \( \tau \) and \( \lambda \).

Proposition 2 shows that subsidies cause households living along the coast to buy more property (intensive margin), which directly increases total property at risk. Further, subsidies cause more households to locate on the coast (extensive margin), which also increases total property at risk. Subsidies including disaster relief, and indeed all parameters governing the insurance market, do not affect the damage distribution of an individual property. Adding property at risk does not change the damage distribution for another property already at risk.

Most of the assumptions are relatively innocuous. For example, we assume independence both across \( h \) and \( d \) and across properties. Adding positive correlation would not, for sufficiently large \( n \), affect the distribution to which \( D \) converges, in the same way that the central limit theorems hold when observations are correlated. The assumption of perfect substitutes implies households own only one type of property, which in turn implies that subsidies affect property demand along the extensive margin. In practice, a small fraction of households owns both coastal (vacation) and inland properties. If coastal and inland properties were not perfect substitutes in the model, households would own both coastal and inland properties, and the effect along the extensive margin would weaken, but the qualitative results would not change.

**Population centers**

We let all property be divided into \( S \) population centers. Following the results from the Coastal population distribution, section we assume each population center \( s \) contains a quantity of property, \( n_s \), drawn from a truncated GPD.\(^{35}\) In particular, we let \( n_s f_{GPD}(n; \sigma, \xi)/f_{GPD}(n; \sigma, \xi) \). Here Eqs. (3) and (4) define \( f_{GPD} \) and \( f_{GPD} \), respectively. If \( \sigma \) satisfies:

\[
n \left( \frac{\sigma}{1 - \xi} - \Gamma(\sigma) \right) = \frac{n}{S}, \tag{28}\]

the mean total property in a population center is \( n/S \) and the mean total amount of property is \( n \).\(^{36}\) The truncation of the distribution ensures that the maximum amount of property in a population center is \( n \).

Population centers have equal geographic area, and are distributed randomly along the coast, which has total area \( J_c \). If a storm path over land covers an area equal to \( J_D \) (determined below), then the probability that a randomly placed population

\(^{35}\) Gabaix (1999) proposes a theory for city sizes that generates a Pareto distribution. In their model, wages adjust to offset city-specific amenity shocks, which keeps the growth rate of cities independent of the size, which implies a power law steady state distribution. Here the property distribution is exogenous, since most of the policies of interest affect total property or the relationship between storm strength and damage, not the property distribution.

\(^{36}\) However, the total amount of property is not equal to \( n \) with probability one. Small differences between \( \sum n_s \) and \( n \) do not affect the results for large \( n \) and scaling the realizations of the population center sizes so that the total property equals \( n \) would vastly complicate the distribution of \( n_i \).
center is in the storm path equals \( \rho = J_D/J_c \). For simplicity, we assume at most one population center is in the storm path.\(^3\) If a population center is in the storm path, then each property in the population center suffers the random loss \( d_i \). Therefore:

\[
\tilde{N} = \begin{cases} 
\frac{n \tilde{F}_{\text{GPD}}(n; \sigma_n, \xi)}{F_{\text{GPD}}(n; \sigma_n, \xi)} & \text{w. p. } \rho \\
1 - \rho & \text{w. p. } 1 - \rho 
\end{cases}
\]  

(29)

\[
\tilde{N} = \begin{cases} 
\tilde{F}_{\text{GPD}}(\tilde{N}; \sigma_n, \xi) / F_{\text{GPD}}(n; \sigma_n, \xi) & \text{if } \tilde{N} > 0 \\
1 - \tilde{F}_{\text{GPD}}(\tilde{N}; \sigma_n, \xi) & \text{if } \tilde{N} = 0 
\end{cases}
\]  

(30)

Therefore \( \tilde{N} \) is a GPD random variable, conditional on a population center being in the storm path.

Eq. (30) implies damage to individual properties exhibits spatial correlation. In particular, the Moran I is positive. However, such spatial correlation does not generate fat tails. If the geographic distribution of properties was thin tailed with spatial correlation, then the central limit theorems still hold and the aggregate damage distribution is thin-tailed. Thus, our approach relies on fat-tailed clustering of property at risk, rather than spatial correlation of individual damages. Monte Carlo models such as FPHLM exhibit spatially correlated damages through spatial correlation in building types and other factors.

**Hurricane properties**

A variety of storm-specific factors influence total damages, including wind speed, the size of the storm, and the path of the storm over land. We model these influences in a simple way. Let \( r \) denote the distance from the center of a storm, with \( t \) the time since landfall. Holland (1980) studies a model of the wind speed, \( w_r(t) \), of a tropical cyclone:

\[
w_r(t) = \frac{r^q}{r^q + \nu} w_r(t) \frac{r^q}{r^q + \nu}
\]  

(31)

Here \( r_m \) is the distance between the center of the storm and the edge of the eye of the tropical cyclone, where the maximum wind speed occurs. Holland (1980) cites estimates of \( q = 0.4, 0.6 \). The wind speed decreases as the distance from the eye of the storm increases.

We assume that building codes and adaptations are such that no damage occurs if the wind speed is less than \( w \). Nordhaus (2010) finds a very high elasticity between wind speed and damages, which he argues could result from building materials that may withstand a maximum amount of wind with little or no damage, and then break all at once when the wind speed crosses a threshold. This motivates our use of a threshold. An alternative is to let adaptations affect the slope of the relationship between damages and wind speed as in Hsiang and Narita (2012), in which case damages would be an increasing function of wind speed. Our threshold model is an accurate approximation if the slope is sufficiently large. The radius of the storm beyond which no damage occurs, \( r \), is:

\[
f(r) = r_m \left( \frac{w_r(t)}{w} \right)^{\frac{q}{2}}
\]  

(32)

Hurricanes lose energy over land, causing wind speed to decline. Following Kaplan et al. (2007), we assume maximum winds decay exponentially with time since landfall:

\[
w_r(t) = w_r(0) \exp(-\delta t).
\]  

(33)

The storm causes no further damage when \( w_r(t) = w \), that is, when the maximum winds are no longer strong enough to cause damage. This occurs at time:

\[
f = \frac{1}{\delta} \log \left( \frac{w_r(0)}{w} \right)
\]  

(34)

Further, combining Eqs. (32) and (33) yields:

---

\(^3\) If instead the assumption was that a storm could intersect multiple population centers, then \( \tilde{N} \) would equal the sum of a random number of GPD random variables. Since the sum of Pareto random variables converges to a stable distribution with the same tail index, the restriction that the storm may intersect only a single population center is unlikely to alter the tail index of \( \tilde{N} \), or the conclusion that \( \tilde{N} \) is fat tailed.
\[ \mathcal{D}(t) = r_m \left( \frac{w_m(0)}{W} \right) \frac{1}{t} \exp \left( -\frac{\delta}{\theta} t \right). \]  

Eq. (35) gives the radius of damage as a function of the wind speed at landfall and the time since landfall. Assume that the storm travels at a constant velocity \( \nu \), which has units of distance, \( m \), per unit of time. Then the area of damage is:

\[ J_D = \int_0^{m(t)} 2\mathcal{D}(t,m)dm = \int_0^{\nu t} 2\mathcal{D}(\frac{m}{\nu})dm, \]

\[ = 2 \int_0^{\nu t} r_m \left( \frac{W}{W} \right) \frac{1}{t} \exp \left( -\frac{\delta}{\nu \theta} m \right)dm. \]  

Here \( \tilde{W} = w_m(0) \), the maximum wind speed at landfall, which we have estimated in the section Distribution of wind speed. Therefore, the probability that the storm intersects a population center is:

\[ \rho = \frac{\int J_D(\tilde{W})}{\int_{\tilde{W}} \mu_{\text{GPD}}(\tilde{W}, \sigma_{\tilde{W}}, \xi_{\tilde{W}})d\tilde{W}.} \]

The empirical evidence that climate change increases the number of tropical cyclones is mixed, but the frequency of category 4 or 5 hurricanes may increase (Bender et al., 2010). Therefore, the model takes \( \sigma_{\tilde{W}} \) as the policy variable, with a decrease in \( \sigma_{\tilde{W}} \) interpreted as the result of climate change abatement. According to the empirical results, storm strength is thin tailed (\( \xi_{\tilde{W}} < 0 \)). The next section derives the distribution of the aggregate damages.

**Theoretical results**

**Fat tails**

The theoretical model has several sources of random variation: storm wind speed, the path of the storm, the quantity of property in an area affected by a storm, and the damage to individual properties in the area affected by a storm. We show here that the distribution for \( \tilde{N} \) largely determines the distribution for the aggregate damages \( D \). Because \( d_i \) is bounded on the unit interval, the variance of \( d_i \) is also bounded. Therefore, the central limit theorems imply that, regardless of the distribution of \( d_i \), if \( \tilde{N} \) is non-stochastic (for example \( \tilde{N} = \beta n \) with \( 0 < \beta < 1 \)), then \( D \) has a normal (thin-tailed) limiting distribution as \( \tilde{N} \to \infty \). Increasing the total property at risk increases the mean and variance of \( D \), but the distribution remains thin tailed.

However, Population centers shows that \( \tilde{N} \) is random. When \( N \) is random, standard central limit theorems no longer hold and the distribution of \( D \) will not in general converge to a normal distribution. Instead, we will show that the limiting distribution of \( D \) will inherit properties of the limiting distribution of \( \tilde{N} \): if \( \tilde{N} \) is thin (fat) tailed, then \( D \) converges to a thin (fat) tailed distribution.

The definition of fat tails poses a difficult problem for modeling the distribution of damages from a natural disaster. As the total number of properties at risk is always finite, for any \( n \) the distribution of damages is bounded and therefore fails to satisfy (2). To reconcile the apparent bound on total damages with empirical tests in Empirical methods, which indicate total damages are fat tailed, we construct a sequence of bounded distributions which converge to a fat-tailed distribution as \( n \to \infty \). For any finite \( n \), the distribution is bounded and not fat tailed, but, as \( n \) becomes large, a fat-tailed distribution becomes an increasingly accurate approximation.

We let \( \tilde{N}_n = \tilde{N} \) to emphasize that \( \tilde{N} \) depends on \( n \). To see how \( D \) inherits the tail properties of \( \tilde{N} \), assume for example, that \( \tilde{N}_n \) has a thin-tailed binomial distribution: \( \tilde{N}_n \text{Binomial} (n, \rho) \) for any \( n \) (as would be the case if each property had a uniform probability of locating at each geographical point on the coast). Then from Robbins (1948), example (ii) as \( n \to \infty \):

\[ D \to^d N[q_k \rho \bar{d}, q_k \rho \bar{d} \left( \sigma^2_{\bar{d}} + (1-\rho)\bar{d}^2 \right)], \]  

Similarly, if \( \tilde{N} \) has a generalized inverse Gaussian distribution, then \( D \) converges to a heavy-tailed (i.e. not fat tailed, but with a decay rate slower than the normal distribution) generalized hyperbolic distribution (Haas and Pigorsch, 2011).

Here, conditional on the storm intersecting a population center, the distribution of \( \tilde{N} \) is a truncated GPD.

**Proposition 3.** Let \( d \mathcal{D} \) be iid with finite mean \( \bar{d} \) and finite variance \( \sigma^2_{\bar{d}} \). Let:
where \( F_{\text{GPD}} \) and \( f_{\text{GPD}} \) are given by (3) and (4), respectively. Then \( D \) converges in distribution to a normal variance mean mixture with Pareto mixing density as \( n \to \infty \). That is:

\[
D \xrightarrow{\text{distribution}} q_{n} D + q_{n} \sigma D \bar{z}.
\]

where \( \bar{z} \) is a standard normal and \( \bar{N} \) is distributed as a GPD with parameters \( \sigma n \) and \( \xi \). Further, \( D \) has tail index \( \alpha = 1/\xi \) and mean:

\[
E \left[ D | \bar{N} > 0 \right] = q_{n} \bar{dN} \left( \frac{\sigma}{1 - \xi} - I \right), \quad \xi < 1, \quad I \equiv \frac{1 - F_{\text{GPD}}(1, \sigma, \xi)}{F_{\text{GPD}}(1, \sigma, \xi)} \frac{1}{1 - \xi}.
\]

Proposition 3 shows that if the number of properties that experience a loss follows a GPD, then for large \( n \), the distribution of damages is approximately a mixture of a normal distribution and a GPD. The form of \( D \) given in Eq. (41) is known as a normal variance mean mixture, with GPD mixing distribution. Proposition 3 shows that \( D \) inherits the tail index of \( N \). The theory is in alignment with the (Empirical results) section, in which the point estimates of the tail index of \( D \) and \( N \) are similar.

In Proposition 3, the number of properties affected is drawn from a distribution bounded by \( n \). At any point in time, the total amount of property remains finite. However, as the total property at risk grows, the bound expands and so does the upper bound for \( \bar{N} \). Because \( \bar{N} \) converges to a fat-tailed distribution, for large \( n \) the distribution of \( \bar{N} \) and \( D \) will appear to have a fat tail, even though for any fixed \( n \) the distribution is bounded and therefore not fat tailed. More precisely, given a fixed, finite sample of tropical cyclones, as \( n \) becomes large, the probability that an observation equals the upper bound approaches zero. Thus, the probability that a bounded and unbounded distribution will fit the data equally well approaches one as \( n \) increases.

It is interesting to compare the mean damages in the model (42), with the mean damages in an ideal setting where property is uniformly distributed (39). The means are not equal unless \( \sigma / (1 - \xi) = 1/S \), as the mixture distribution exhibits skewness while the normal distribution is symmetric. More importantly, because uniformly distributed property results in damages being drawn from a thin-tailed normal distribution, catastrophic storms in our model cause far more damage.

The expected damage from a catastrophic storm is implicit from the distribution of damages given by Proposition 3. Because \( \bar{z} \) and \( \bar{N} \) are independent, Eq. (41) implies that \( D^* \) is the solution to:

\[
\text{Prob}(D \geq D^*) = \frac{1}{T},
\]

\[
\text{Prob}(D \leq D^*) = 1 - \frac{1}{T},
\]

\[
1 - p + \rho \text{Prob}(D \leq D^* | \bar{N} > 0) = 1 - \frac{1}{T},
\]

\[
F_{\text{D}}(D) = \text{Prob}(D \leq D^* | \bar{N} > 0) = 1 - \frac{1}{\rho T},
\]

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{z(\bar{N}, D^*)} f_{\bar{z}}(\bar{z}) d\bar{z} \bar{N} \left( \frac{\bar{N}}{\bar{N}} \right) d\bar{N} = 1 - \frac{1}{\rho T},
\]

where \( f_{\bar{z}} \) is the standard normal density, \( f_{\bar{N}} \) is the GPD with parameters \( \sigma n \) and \( \xi \), and:

\[
z(\bar{N}, D^*) = \frac{D^* - q_{n} \bar{dN}}{q_{n} \sigma_d \bar{N}^2}.
\]

Eq. (48) implicitly solves for \( D^* \) so that the mass of the tail is one percent for \( T = 100n_0 \). If a change in a parameter value results in a higher value of \( D^* \), then the change in the parameter adds mass to the tail (but it does not affect the rate of decay of

38 See Haas and Pigorsch (2011) for a discussion of normal variance mean mixtures.
the tail, which is always governed by $\xi$, as shown in Proposition 3. Eq. (48) shows that $D^*$ and the mass of the tail depends on all subsidies, $\sigma$ and $\xi$.

From Eqs. (42), (47), and (48), we see that the policy variables affect both the mean damage and the damage from a $T$ observation storm in different ways. We examine these effects in the next section.

**Policy implications**

Four policy variables exist in the model. First, the government may set the disaster relief policy $\tau$. Second, the government can subsidize the cost of coastal property by reducing $q_w$. Third, the government can choose the relative strength of adaptations, $W$. Fourth, the government could, through climate change regulation, attempt to reduce the mean or the mass of the upper quantiles of the storm strength distribution.

**Proposition 4.** Let the assumptions of Proposition 3 hold, and assume $\rho > 0$. Then a decrease in the disaster relief subsidy ($\tau$), a decrease in the property subsidy ($\lambda$), an increase in the strength of adaptations ($W$), and a decrease in the scale parameter of the storm strength distribution ($\sigma_w$) all cause a decrease in the damage from a $T$ observation storm ($D^*$) and a decrease in the mean damages (E[D]). The tail index of the damage distribution is not affected by $\tau$, $\lambda$, $W$, or $\sigma_w$.

While each policy variable affects the mean and catastrophic damages, the channels differ substantially. First, a decrease in the disaster relief subsidy reduces the total property at risk, through both the intensive and extensive margins. In turn, a decrease in the total property at risk has two effects. First, each population center has less property at risk, causing the mean damage ($n/S$) to decrease. Second, large population centers have less property at risk, so that large damages will occur less often.

To ensure that a catastrophic storm occurs only once every $T$ observations, the amount of damage required for a storm to be considered catastrophic must increase. Second, an increase in the total property at risk extends the right tail of the truncated GPD. However, this effect vanishes for large $n$, because as $n$ increases, a realization of $N$ at the truncation point becomes less and less likely. That is, as the number of properties increases, the likelihood that a storm affects all coastal property at risk approaches zero.

As with the disaster relief subsidy, the property subsidy increases the total property, along both the intensive and extensive margins. Therefore, a decrease in the property subsidy decreases the damage from all storms, including catastrophic storms.

An increase in the strength of adaptations, $W$, reduces the geographic area of damage in two ways. First, the radius of winds which are strong enough to cause damage decreases, as the wind speed at a certain range of distances from the eye are no longer strong enough to cause damage. Second, the time since landfall at which the storm causes no further damage decreases, because $W$, the minimum wind speed which causes damage, increases. In turn, the reduction in the geographic area of damage implies that fewer storms will intersect a population center and cause damage. Therefore, the unconditional mean damage decreases. In addition, in a sample of $T$ storms, fewer storms cause damage, meaning that there will be fewer storms that cause damage greater than a given level, $D$. So, $D^*$ must decrease to bring the unconditional probability of a storm causing damage greater than $D^*$ back to $1/T$. Alternatively, in a 100-year sample, fewer storms cause damage, so fewer storms are catastrophic, which decreases the likelihood that a catastrophic storm causes damage above a given level.

We model climate change abatement as a decrease in the scale parameter of the wind distribution, which results in storms with lower (in the sense of first order stochastic dominance) wind speeds. As a result, the geographic area of damage decreases both because the radius of the tropical cyclone is smaller and because the storm dissipates more quickly. Climate change abatement reduces the number of storms that intersect a population center, reducing both the mean damage and the damage from a catastrophic storm.

**Proposition 3** indicates that disaster relief policy and the structure of the insurance market increase the mean of $D$ by increasing $n$. However, disaster relief and the structure of the insurance market do not affect the tail index, $\alpha$, which determines the tail behavior of $D$. Similarly, adaptation and climate change abatement can affect the probability that a storm causes damage, but do not affect the tail index.

**Proposition 4** shows that policies exist which decrease the mass of the tail, albeit leaving the tail index unaffected. However, a policy which reduces the mass of the tail does not necessarily increase welfare. For example, the benefits of adaptation and climate abatement policies must be weighed against their costs, and the benefits are affected by pre-existing subsidies. The next proposition formalizes these trade-offs.

**Proposition 5.** Let the assumptions of Proposition 4 hold. Assume further that utility is logarithmic and households are weighted equally in the social welfare function:

$$W = \int_{0}^{1} \left( \log |c_i| + \log [\phi(i)A_{Ki} + A_{W}] \right) di.$$  \hspace{1cm} (49)

Then a decrease in the disaster relief subsidy ($\tau$) and a decrease in the property subsidy ($\lambda$) result in an increase in welfare. Suppose also that $\tau$ and $\lambda$ are sufficiently small. An increase in the strength of adaptation $\sigma$ and a decrease in the scale parameter of the storm strength distribution ($\sigma_w$) increase welfare if the cost of changing these parameters is sufficiently small.
The subsidies \( r \) and \( \lambda \) do not correct any externality, but instead distort the property decision away from the social optimum (such subsidies are known as “perverse subsidies,” see Bagstada et al., 2007). Thus, reducing these subsidies corrects pre-existing distortions (too much coastal property and too many households living on the coast), resulting in a welfare increase, while at the same time decreasing the mass of the fat tail.

Surprisingly, directly reducing expected storm damages either by increasing adaptations or by reducing storm strength (through climate change abatement) is not necessarily beneficial if substantial pre-existing distortions exist. A decrease in expected damages encourages more households to move to the coast, which in turn increases the number of subsidized households, which reduces welfare. Further, a decrease in expected damages may increase the overall property subsidy rate (equal to the after subsidy price \( q_K(1 - \lambda + (1 - r)h_d) \) divided by the gross price \( q_K(1 + h_d) \)), further decreasing welfare. But if the pre-existing distortions are small, then reducing expected damages produces a welfare benefit. Changes in policies which reduce damages create real income transfers by reducing the gross price of coastal property. When subsidies are assumed small in Proposition 5, such income effects are also small, and the effect of a policy change on welfare is determined only by the effect of the policy on damages.

The model does not include any costs of adaptations or climate change abatement. Proposition 5 shows only the benefits associated with such policies (if the pre-existing subsidies are small), which must still be weighed against the cost of the policy in a benefit-cost analysis. We focus here only on the potential benefits of these policies, because policy makers have better access to the cost of adaptations and climate abatement policies.39

Finally, note that many costs associated with fat-tailed distributions are not modeled in the above welfare analysis. First, the difficulty of estimating the damage from a catastrophic storm means that households may form incorrect beliefs, and reduce insurance coverage. Further, insurance firms must hold more costly reserves due to the possibility of catastrophic storms. The cost of holding reserves is passed on, in the form of higher premiums, to policyholders, who may respond by not fully insuring (Kousky and Cooke, 2012). This outcome not only exposes the household to risk, but reduces the risk pool available to insurance firms. Including such costs would only increase the welfare gains derived in Proposition 5.

Conclusion

Tropical cyclones can impose significant welfare losses on coastal communities. The empirical analysis shows that tropical cyclone damages from 1900—2012 follow a fat-tailed distribution, so much so that the parameter estimates indicate infinite variance. Estimates of catastrophic damages are larger, with wider standard errors, when damages follow a fat-tailed distribution as opposed to a thin-tailed distribution. Insurers facing fat-tailed risk must hold more reserve capital. The cost of holding reserves is passed on to policyholders in the form of higher premiums.

A theoretical model of homeowner behavior, population centers, and storm strength is presented. The model reveals that homeowners will not fully insure their properties from risk of damage, only paying premiums to recoup losses not covered by the disaster-relief program. In this way, the disaster relief program is equivalent to an insurance premium subsidy, which tends to increase the quantity of coastal property along both the extensive and intensive margins. A subsidy to the cost of coastal property also increases the quantity of coastal property along both margins. These subsidies increase the mean damage and the damage from a catastrophic storm. However, subsidies do not create a fat tail. Surprisingly, the fat tail results from the distribution of households in population centers. The empirical results agree with the prediction of the theoretical model that the tail index of the population center distribution equals the tail index of the damage distribution.

In the model, adaptations and climate change abatement offer the possibility to reduce the coastal area over which winds are strong enough to result in damage. More storms miss population centers, which lowers the average damage and also implies storms intersect large population centers less often.

A natural question for future research is applying our model to other fat-tailed empirical damage distributions, such as flood damages. Important differences exist between cyclone and flood damages, including the nature of the insurance market (for example, flood insurance is publicly provided) and that flood damage to individual properties is more spatially correlated than wind damage. Regardless, both types of damage inherently depend on the spatial distribution of property. Unlike wind speed, Nordhaus (2011) finds that earthquake strength is fat tailed. Nonetheless, the distribution of property also likely plays a role in the earthquake damage distribution. It would be surprising if the mechanism we have identified — the fat tail in the property distribution — did not play an important role in the distribution of damages from other disasters.

A theoretical and empirical examination of the effect of zoning laws on the coastal property distribution, and ultimately the damage distribution would also be an interesting extension. Zoning laws may restrict property accumulation along the extensive margin by prohibiting development in coastal areas. In addition, zoning laws could restrict property accumulation along the intensive margin by, for example, limiting the size of houses built in coastal areas.

Another implication of our results is that previous catastrophic storms are poor predictors of the damage from future catastrophic storms. Thus, our model predicts a catastrophic storm can quickly change public risk perceptions. Whether or not changes in behavior following a catastrophic storm, such as purchasing more insurance, are consistent with updating a fat-

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39 For example, the EPA’s social cost of carbon framework estimates the impact (including hurricane impact) of the emission of a ton of carbon. The EPA then approves regulations which cost less per ton than the social cost of carbon (even though most abatement policies undertaken by the US will likely have only a minimal near term effect on the global stock of carbon, which has built up over centuries and depends on the collective actions of all countries).
tailed risk distribution, is a subject of future research. Terry (2017) finds that mean hurricane damages in the United States will increase dramatically by 2075, due to climate change and coastal development. As the population and property agglomerate in coastal areas, the importance of catastrophic storms will also only increase in the future. Thus, the need for policies that can minimize the occurrence of, or damage from, catastrophic storms will only become more acute.

Appendix A. Proof of Theorems

A.1. Proof of Proposition 1

The Kuhn-Tucker conditions for problem (23) require $\mu_{ji}A_{ji} = 0$, where $\mu_{ji}$ is the multiplier which ensures property choices are non-negative and $j \in [K,I]$. Eliminating the multipliers from the first order conditions for optimal property choice results in:

$$(-w|c_i|q_K\left(1 - \lambda + (1 - \tau)\bar{h}d\right) + \phi(i)v'[\phi(i)A_{K,i} + A_{I,i}])A_{K,i} = 0,$$

$$(-w|c_i|q_I + v'[\phi(i)A_{K,i} + A_{I,i}])A_{I,i} = 0.$$ (51)

We first show that $A_{K,i} > 0$ implies $A_{K,i} = 0$ for all $i \neq i^*$. Since $A_{I,i} > 0$, $\mu_{I,i} = 0$. Therefore, Eq. (51) reduces to:

$$w|c_i|q_I = w[\phi(i)A_{K,i} + A_{I,i}].$$ (52)

Combining (52) and (50) results in:

$$(-w|c_i|q_K\left(1 - \lambda + (1 - \tau)\bar{h}d\right) + w|c_i|\phi(i)q_I)A_{K,i} = 0.$$ (53)

Therefore either $A_{K,i} = 0$ or:

$$\phi(i) = q_r\left(1 - \lambda + (1 - \tau)\bar{h}d\right).$$ (54)

However, since the right hand side of (24) is independent of $i$ and $\phi$ is strictly increasing, Eq. (24) holds for at most one $i$, denoted $i^*$. Therefore, for all $i \neq i^*$ for which $A_{K,i} > 0$, we have $A_{K,i} = 0$. Reversing the argument implies that $A_{K,i} > 0$ implies $A_{I,i} = 0$ for all $i \neq i^*$. For the converse, we must show that $A_{K,i} = 0$ implies $A_{I,i} > 0$. Suppose not, suppose $A_{K,i} = A_{I,i} = 0$. Then total housing is zero which cannot hold by the Inada conditions. Therefore $A_{K,i} = 0$ if and only if $A_{I,i} > 0$ and the reverse.

Next from (52) and (53), the range of $i$ for which $A_{K,i} = 0$ and $A_{I,i} > 0$ satisfies:

$$w|c_i|q_K\left(1 - \lambda + (1 - \tau)\bar{h}d\right) \geq w|c_i|\phi(i)q_I,$$ (55)

$$\phi(i) \leq q_r\left(1 - \lambda + (1 - \tau)\bar{h}d\right).$$ (56)

Since $\phi(i)$ is increasing, the above inequality is satisfied for all $i < i^*$, where $i^*$ satisfies (24). Reversing the argument shows that $A_{K,i} > 0$ and $A_{I,i} = 0$ for all $i \geq i^*$.

It remains to show that $i^* \in (0,1)$. Since $\phi(0) = 0$ by assumption, condition (56) holds with strict inequality for $i = 0$. Therefore $i^* > 0$. Since $\phi(1) > 2q_r$ and $\bar{h}$, $\bar{d}$, and $\bar{d}$ are all between zero and one, condition (56) is not satisfied for $i = 1$. Therefore $i^* < 1$.

A.2. Proof of Proposition 2

Propositions 2.1, 2.2, and 2.5 follow from applying the implicit function theorem to Eqs. (24), (25), and (26) respectively. For 2.3, note that:

$$qKn = q_K \int_{\tau_{i^*}}^1 A_{K,i}(\tau) d\tau.$$ (57)

Applying Leibniz’s rule results in:
\[
\frac{\partial q_K n}{\partial \tau} = q_K \left( \int_0^1 \frac{\partial A_{K,i}}{\partial \tau} di - A_{K,i} \frac{\partial i^*}{\partial \tau} \right)
\] (58)

The result then follows since Propositions 2.1 and 2.2 show that \( i^* \) is decreasing in \( \tau \) and \( A_{K,i} \) is increasing in \( \tau \).

For 2.4, for the special case where both \( u \) and \( v \) are logarithmic, Eq. (25) reduces to:

\[
\frac{q_K (1 + (1 - \tau)h d)}{w - T - q_K A_{K,i} (1 + (1 - \tau)h d)} = \frac{1}{A_{K,i}}.
\] (59)

Solving for \( A_{K,i} \) results in Eq. (27).

A.3. Proof of Proposition 3

The following lemma is Theorem 3 from Peter and Clark (1973), which we will apply directly.

**Lemma 6.** Let \( d_i u \) be iid with mean zero and unit variance and \( N_n \) be a random variable independent of \( d_i \) such that:

\[
\frac{\tilde{N}_n}{n} \to^p \tilde{M} \text{ as } n \to \infty.
\] (60)

Then:

\[
D = n^{-\frac{1}{2}} \sum_{i=1}^{N_n} d_i \to^d f_D,
\] (61)

where \( f_D \) is the distribution of a normal variance mean mixture. That is: \( D \to \tilde{M}^2 \tilde{z} \), where \( \tilde{z} \) is distributed as a standard normal.

The proof applies Lemma 6. First, note that \( \tilde{M} \equiv \tilde{N}_n / n \) has distribution:

\[
\frac{\tilde{N}_n}{n} \to^p \tilde{M} \text{ as } n \to \infty.
\] (62)

\[
= \frac{1}{\sigma_N} \left( 1 + \frac{\xi}{n \sigma_N} \left( \tilde{M} \right) \right)^{-\frac{1}{2}} n,
\] (63)

\[
= \frac{1}{\sigma_N} \left( 1 + \frac{\xi}{\sigma_N} \tilde{M} \right)^{-\frac{1}{2}},
\] (64)

which is independent of \( n \). Therefore, \( \tilde{N}_n / n \) converges trivially to \( \tilde{M} \).

The next step converts the problem into one that satisfies the assumptions of Lemma 6. Let \( y_i = (d_i - \bar{d}) / \sigma_d \), then by Lemma 6, conditional on \( \bar{N} \),

\[
n^{-\frac{1}{2}} \sum_{i=1}^{\tilde{N}} y_i \to^d N \left[ 0, \tilde{M} \right],
\] (65)

\[
n^{-\frac{1}{2}} \left( \sum_{i=1}^{\tilde{N}} d_i - \bar{d} \tilde{N} \right) \to^d N \left[ 0, \sigma_d^2 \tilde{M} \right].
\] (66)

Therefore for \( n \) large:

\[
\frac{\partial q_K n}{\partial \tau} = q_K \left( \int_0^1 \frac{\partial A_{K,i}}{\partial \tau} di - A_{K,i} \frac{\partial i^*}{\partial \tau} \right)
\] (58)
\[
\sum_{i=1}^{N} d_i - \bar{d}N \xrightarrow{d} N \left[ 0, \sigma^2_{d \bar{N}} \right], (67)
\]

\[
\sum_{i=1}^{N} d_i \xrightarrow{d} N \left[ \bar{d}N, \sigma^2_{d \bar{N}} \right], (68)
\]

\[
D \xrightarrow{d} N \left[ qK\bar{d}N, q^2K\sigma^2_{d \bar{N}} \right], (69)
\]

which is the normal variance mean mixture (41) with the parameters given in the proposition. Since the limit of \( \bar{N}_n \) is the GPD, the mixing distribution is the GPD.

The moments and tail behavior follow from the properties of the normal variance mean mixture.

**Lemma 7.** Let \( D \) be a normal variance mean mixture, i.e.:

\[
D = \mu \bar{N} + \sigma \bar{N}^2 \bar{z},
\]

where \( \bar{z} \) is a standard normal independent of \( \bar{N} \), and \( \bar{N} \) has finite mean. Then:

1. \( E[D] = \mu E[\bar{N}] \).
2. The tail index of \( D \) equals the tail index of \( \bar{N} \).

We prove each result separately.

1. Since \( \bar{N} \) and \( \bar{z} \) are independent:

\[
E[D] = \mu E[\bar{N}] + \sigma E[\bar{N}^2]E[\bar{z}].
\]

Next Jensen’s inequality implies \( E[\bar{N}^2] \leq E[\bar{N}]^2 < \infty \). Therefore, since \( \bar{z} \) has mean zero, the second term in (71) is zero and the result follows.

2. The \( k \)th moment of \( D \) is finite if and only if:

\[
E[D^k] = E \left[ \left( \mu \bar{N} + \sigma \bar{N}^2 \bar{z} \right)^k \right] < \infty. (72)
\]

The binomial theorem implies:

\[
E[D^k] = \sum_{i=0}^{k} \binom{k}{i} \left( \frac{1}{\bar{d}N} \right)^{k-i} \left( \frac{\sigma^2_{d \bar{N}}}{\bar{N}^2} \right)^i. (73)
\]

A property of the GPD is that if the \( i \)th moment exists, then so do all moments \( j \leq i \). The largest exponent on \( \bar{N} \) in (73) occurs when \( i = 0 \). Therefore it sufficient to show \( E[\bar{N}^k] \) exists. But since \( \bar{N} \) has tail index equal to \( \alpha \), \( E[\bar{N}^k] \) exists if and only if \( \alpha > k \). Therefore the \( k \)th moment of \( D \) exists if and only \( \alpha > k \). Therefore, \( \alpha \) is the tail index of \( D \). \( \square \)

**A.4. Proof of Proposition 4**

Eqs. (30) and (42) imply the unconditional mean damage is:
\[ D = \rho q_k \overline{a_n} \left( \frac{\sigma}{1 - \zeta} - \Gamma \right). \]  

(74)

From Eq. (74), the unconditional mean damage is increasing in \( n \). Further, Proposition 2.3 shows that \( n \) is increasing in \( \tau \) and \( \lambda \). Therefore, \( E[D] \) is increasing in \( \tau \) and \( \lambda \). The unconditional mean damage is also increasing in \( \rho \). Further, Eqs. (37) and (38) imply that \( \rho \) is decreasing in \( \varpi \). Therefore, \( E[D] \) is decreasing in \( \varpi \). Finally, for \( \sigma_w \), combining Eqs. (37) and (38) gives:

\[ \rho = \int_0^2 \frac{2r_m \theta_0}{J_c} \left( \frac{\varpi}{\varpi} \right) \exp \left( \frac{\delta}{\nu^0} m \right) \text{dms} \]  

\[ \rho = \int_0^2 \frac{2r_m \theta_0}{J_c} \left( \frac{\varpi}{\varpi} \right) - 1 \right) f_{\text{GPD}} \left( \varpi, \sigma_w, \xi_w \right) d\varpi = \int_0^2 h \left( \varpi \right) f_{\text{GPD}} \left( \varpi, \sigma_w, \xi_w \right) d\varpi. \]  

(75)

Note that \( f_{\text{GPD}} \) is decreasing in \( \sigma_w \) and therefore \( \bar{N}_1 F_{\text{GPD}}(\sigma_{w1}) \) first order stochastic dominates \( \bar{N}_2 F_{\text{GPD}}(\sigma_{w2}) \) for \( \sigma_{w1} > \sigma_{w2} \). Therefore, since \( h \) is an increasing function, \( \rho \) is increasing in \( \sigma_w \). Therefore, the unconditional mean damage is increasing in \( \varpi \).

For the damage from a \( T \) observation storm, Eq. (47) defines an implicit function \( D^* (\varpi, \tau, \lambda, \sigma_w) \). Eq. (47) simplifies to:

\[ \int_0^\infty F_z \left( \frac{D^* - q_k a_N}{q_k \sigma_d N^2} \right) f_N \left( \bar{N}, \sigma_n \right) d\bar{N} = 1 - \frac{1}{\rho T}. \]  

(77)

Consider first the derivative with respect to \( n \). Taking the total derivative of (77) yields:

\[ \int_0^\infty F_z \left( \frac{D^* - q_k a_N}{q_k \sigma_d N^2} \right) \frac{\partial D^*}{\partial \varpi} f_N \left( \bar{N}, \sigma_n \right) d\bar{N} + \int_0^\infty F_z \left( \frac{D^* - q_k a_N}{q_k \sigma_d N^2} \right) \frac{\partial f_N}{\partial \varpi} d\bar{N} = 0. \]  

(78)

\[ \frac{\partial D^*}{\partial \varpi} = - \int_0^\infty F_z \left( \frac{D^* - q_k a_N}{q_k \sigma_d N^2} \right) \frac{\partial f_N}{\partial \varpi} d\bar{N}. \]  

(79)

Next, the derivative of the GPD with respect to \( n \) follows from Eq. (4):

\[ \frac{\partial f_N}{\partial \varpi} \left( \bar{N}, \sigma_n \right) = \bar{N} - \sigma_n \left( 1 + \frac{\zeta}{\sigma_n} \right)^{-2 - \frac{1}{\zeta}}. \]  

(80)

Therefore:

\[ \int \frac{\partial f_N}{\partial \varpi} \left( \bar{N}, \sigma_n \right) d\bar{N} = \bar{N} - \sigma_n \left( 1 + \frac{\zeta}{\sigma_n} \right)^{-1 - \frac{1}{\zeta}} = \frac{\bar{N} - \sigma_n}{n f_N \left( \bar{N} \right)} d\bar{N}. \]  

(81)

Next, the numerator of (79) simplifies using (81) and integration by parts:

\[ - \int_0^\infty F_z \left( \frac{D^* - q_k a_N}{q_k \sigma_d N^2} \right) \frac{\partial f_N}{\partial \varpi} d\bar{N} = - F_z \left( \frac{D^* - q_k a_N}{q_k \sigma_d N^2} \right) \frac{N}{n f_N \left( \bar{N} \right)} \bigg|_0^\infty + \int_0^\infty F_z \left( \frac{D^* - q_k a_N}{q_k \sigma_d N^2} \right) \frac{\partial f_N}{\partial \varpi} \frac{N}{n f_N \left( \bar{N} \right)} d\bar{N}. \]  

(82)
Simple calculations show the first term on the right hand side of (82) is zero at both integration end points. The derivative (79) thus simplifies to:

\[
\frac{\partial D^*}{\partial n} = \frac{\int_0^\infty f_z \left( \frac{D^* - q_k \bar{d}N}{q_k \sigma_d \bar{N}^2} \right) \frac{\partial z}{\partial D^*} f_N \left( \bar{N} \right) d\bar{N}}{\int_0^\infty f_z \left( \frac{D^* - q_k \bar{d}N}{q_k \sigma_d \bar{N}^2} \right) \frac{\partial z}{\partial N} f_N \left( \bar{N} \right) d\bar{N}}.
\]  

Both the numerator and denominator of (84) are positive, and so the derivative is positive. Proposition 2.3 shows that \( n \) is increasing in \( \tau \) and \( \lambda \). Therefore, \( D^* \) is increasing in \( \tau \) and \( \lambda \).

Consider now the derivative of \( D^* \) with respect to \( r \). Following a similar logic, Eq. (77) implies:

\[
\frac{\partial D^*}{\partial r} = \frac{\int_0^\infty f_z \left( \frac{D^* - q_k \bar{d}N}{q_k \sigma_d \bar{N}^2} \right) f_N \left( \bar{N} \right) \bar{N}^{-\frac{1}{2}} \left( D^* + q_k \bar{d}N \right) \frac{1}{2n} d\bar{N}}{\int_0^\infty f_z \left( \frac{D^* - q_k \bar{d}N}{q_k \sigma_d \bar{N}^2} \right) f_N \left( \bar{N} \right) \bar{N}^{-\frac{1}{2}} d\bar{N}}.
\]  

A.5. Proof of Proposition 5

Evaluating Eqs. (25) and (26) assuming logarithmic utility results in:

\[
A_{ki} = \frac{\omega - T}{2q_l}, \quad i < i^*, \quad i = 1,
\]

\[
A_{ki} = \frac{\omega - T}{2q_K}, \quad i \geq i^*, \quad \hat{q}_K \equiv q_k \left( 1 - \lambda + (1 - \tau) \bar{h} \bar{d} \right),
\]

\[
c_i = \frac{\omega - T}{2}, \quad \forall i.
\]

The decisions and Eq. (49) imply the social welfare is:

\[
W = \int_0^1 \log \left[ \frac{\omega - T}{2} \right] di + \int_0^i \log \left[ \frac{\omega - T}{2q_l} \right] di + \int_{i^*}^1 \log \left[ \Phi(i) \frac{\omega - T}{2q_K} \right] di.
\]  

Since the property and consumption decisions are identical across coastal households and the same for inland households:

\[
W = 2 \log \left[ \frac{\omega - T}{2} \right] - i^* \log [q_l] - (1 - i^*) \log [\hat{q}_K] + \int_{i^*}^1 \log [\Phi(i)] di.
\]  

We next compute the equilibrium tax. The government budget constraint (17) sets tax revenue equal to subsidy payments. Given that \( n = \int_0^1 A_{ki} di \), we have:
welfare effect is ambiguous.

Given Eq. (87), the budget constraint simplifies to:

\[
T = (1 - i') \left( \frac{\omega - T}{2} \right) \frac{\eta d}{\lambda + \tau h d}. \tag{92}
\]

It will be convenient to introduce notation. Let \( 1 - y \) denote the ratio of the subsidized to the gross price of \( A_{K_i} \) (inclusive of the insurance cost):

\[
1 - y = 1 - \lambda + (1 - \tau) \frac{\eta d}{1 + \eta d}, \tag{93}
\]

\[
y = \lambda + \tau \frac{\eta d}{1 + \eta d}. \tag{94}
\]

Thus, \( y \) is the subsidy as a fraction of the gross price, and:

\[
T = (1 - i') \left( \frac{\omega - T}{2} \right) \frac{\eta d}{1 - y}, \tag{95}
\]

\[
T = \frac{\omega(1 - i') y}{2 - y(1 + i')} \tag{96}
\]

\[
\frac{\omega - T}{2} = \frac{\omega(1 - y)}{2 - y(1 + i')} \tag{97}
\]

Welfare is then:

\[
W = 2 \log \left[ \frac{\omega(1 - y)}{2 - y(1 + i')} \right] - (1 - i') \log \left[ 1 + \frac{\eta d}{\lambda + \tau h d} \right] - i' \log |q_i| \\
- (1 - i') \log |q_{K_i}| -(1 - i') \log [1 - y] + \int_0^1 \log(\phi(i))di. \tag{98}
\]

\[
= 2 \log |\omega| - 2 \log [2 - y(1 + i')] - (1 - i') \log \left[ 1 + \frac{\eta d}{\lambda + \tau h d} \right] - i' \log |q_i| -(1 - i') \log |q_{K_i}| + (1 + i') \log [1 - y] + \int_0^1 \log(\phi(i))di. \tag{99}
\]

Next, welfare is decreasing in either subsidy \( s = \lambda, \tau \) if and only if:

\[
\frac{\partial W}{\partial s} = \left. \frac{2y}{2 - y(1 + i')} + \log \left[ 1 + \frac{\eta d}{\lambda + \tau h d} \right] - \log |q_i| + \log |q_{K_i}| + \log [1 - y] - \log(\phi(i)) \right\} \frac{\partial i'}{\partial s} + \left. \frac{2(1 + i')}{2 - y(1 + i')} \right. - \frac{1 + i'}{1 - y} \frac{\partial y}{\partial s} \tag{100}
\]

Exploiting Eq. (24) simplifies the derivative to:

\[
\frac{\partial W}{\partial s} = \frac{2y}{2 - y(1 + i')} \frac{\partial i'}{\partial s} - \frac{(1 + i')(1 - i')y}{(2 - y(1 + i'))(1 - y)} \frac{\partial y}{\partial s} \tag{101}
\]

Finally, using the implicit function theorem, Eq. (24) implies \( i' \) is decreasing in both subsidies. Thus, the first term of (101) is negative. Further, \( y \) is increasing in both subsidies so the second term is negative as well. Thus welfare is decreasing in both subsidies.

Consider now how welfare changes in response to changes in \( \omega \) and \( \sigma \). An analogous argument as above implies:

\[
\frac{\partial W}{\partial \omega} = \left. \frac{2y}{2 - y(1 + i')} \right. \frac{\partial i'}{\partial \omega} - \frac{(1 + i')(1 - i')y}{(2 - y(1 + i'))(1 - y)} \frac{\partial y}{\partial \omega} - \frac{1 - i'}{1 + \eta d} \tag{102}
\]

However, the derivative of \( i' \) with respect to \( \eta d \) is positive and the sign of the derivative of \( y \) is also positive, so the overall welfare effect is ambiguous.
Hence, welfare is decreasing in $\frac{\sigma'}{n}$ if the subsidies are sufficiently small. Finally, from Eq. (42):

$$\frac{\partial W}{\partial d} = \rho \left[ \frac{\sigma}{1 - \sigma} - \frac{I}{n} \right].$$

Further, the proof of Proposition 4 establishes $\rho$ is increasing in $\sigma$ and $E[\varphi]$ is decreasing in $\varphi$. Therefore, if the subsidies are sufficiently small, an increase in $\varphi$ and a decrease in $\sigma$, increase welfare.

References


